Design Loop Challenge
It’s all about pitch!

Design 3 “musical” instruments, each constructed to produce an assigned note within specifications. You need to build these instruments from scratch and not just for example use an existing instrument. You need to do more than a computer simulation. There are a number of possible approaches and your challenge is to choose the approaches that interest your team and go through the design loop process to produce the assigned frequencies within specifications.

Suggestions: Try to “keep it simple” and work within the interests and capabilities of your team.

The products you will design and produce are three instruments, each producing a different one of the assigned three notes (to within 3%) and also exploiting a different physical process to do this. Your instruments must produce a sound level of at least 70 db.

A report will be written summarizing the theory used and documenting the iterative design loop processes you went through to achieve your goal. There should be plots comparing theory with the final frequencies obtained. The report should discuss possible sources of error and contain drawings and/or photographs of your completed devices.

A power point presentation will be made summarizing your results and demonstrating your instruments.

The teams will be combined into an orchestra and play a piece of music as the finale to this project.

Due Date: Wednesday, September 19

The following section outlines a number of possible approaches for designing your instruments. For many of these, equations are given to guide your designs. Remember you need to base your designs in theory to guide your way through the design loop.
Possible Instruments

There are many possible physical phenomena to use to design your instruments. Many of these are applied in common musical instruments. Can you think of other choices?

a. Wind Driven Instruments

- Helmholtz resonators
- Corrugated pipes (The magic flute)
- Organ pipes
- Aeolian harps
- Balloon Sounds (Bagpipes or---)
- Whistles
- Horns

b. Friction/ Rubbing Instruments

- Wine glass
- Squeaking door
- Balloon
- Chalkboard squeaks

c. Percussion

- Membrane
- Drum
- Bell
- Gong

d. Plucking or forcing

- String under tension

What other possibilities can you think of?
e.g. An electric circuit of your own design driving a speaker
Some Useful Relationships

1. Helmholtz Resonators

A common Helmholtz resonator is a liter bottle of soft drink that sounds a note as you blow across the top. You may have noticed that the tone changes as you finish off more and more of the drink. The resonator was named after the well-known acoustics scientist.

![Diagram of a Helmholtz Resonator]

The volume does not have to be spherical but should be much larger than the neck volume. This system is an analog of a spring/mass system. The air in the neck is the mass in this case and the compressibility of the air in the volume the spring.

The equation predicting the resonant frequencies of the system is

\[ F = \frac{c}{2\pi} \left( \frac{S}{L V} \right)^{1/2} \]
where: $F$ is the frequency,
$c$ is the speed of sound,
$V$ is the volume,
$S$ is the area of the neck (typically $\pi a^2$), and
$L_c$ is the corrected length of the neck.

$L_c = L + \frac{16a}{2\pi}$, where $L$ the neck length and $a$ is the neck radius.

Examples of Helmholtz resonators are bottles and containers of various kinds. Also, the sound some people can make when blowing through cupped hands is probably a result of this type of resonance.

2. Corrugated Tubes

Blowing continuously through a smooth tube produces no noticeable tones. However, when a tube is corrugated, as are some straws for water bottles, a strong tone can result. This effect has been used in operas with dancers twirling the tubes to produce internal flows and sound. A toy has been constructed based upon this phenomenon.

Scientific studies have been done in an effort to understand the physics (e.g. F.S. Crawford, Singing Corrugated Pipes, American Journal of Physics, 42, pp278-288, 1974). You can model this in part by considering the tube to be an organ pipe open at both ends. However, you also need to model the process causing the pipe to sound. This can be done by determining the frequencies generated by the flow in the pipe passing over the corrugations and then estimating the conditions under which these frequencies become matched.

Thus for the organ pipe:

\[ F = \frac{Nc}{2L} \]

Where $N$ defines the fundamental and harmonics with integers 1, 2---
$c$ is the speed of sound, and
$L$ is the pipe length.

The corrugation frequency can be estimated by taking the ratio of the estimated flow speed in the tube to the corrugation spacing.
F_{corrugation}= U/d, where U is the estimated flow speed, and d is the distance between corrugations. As the flow speed increases or the corrugation spacing decreases the frequency generated may be expected to increase. The loudest sound should occur when the corrugation frequency matches one of the organ pipe resonances.

3. Organ Pipes

The expression for the resonant frequency for an organ pipe open at both ends is:

\[ F = \frac{Nc}{2L} \]

Where N defines the fundamental and harmonics with integers 1, 2, ..., c is the speed of sound, and L is the pipe length.

If the pipe is closed at one end the expression is

\[ F = \frac{(2N-1)c}{4L} \]

For an instrument consisting of a hollow tube blown at one end (such as a didgeridoo) these relationships could be used as working models to estimate the frequencies produced.

4. Vibrating Strings

For a clamped/clamped string the frequencies may be computed from

\[ F = \frac{NCs}{2L} \]

Where:  N is an integer defining the fundamental and harmonics
L is the string length
Cs is \((T/\rho)^{1/2}\)
and T is the tension (e.g. in Newtons)
\(\rho\) is the linear density of the string (e.g. kilograms per meter)

6. Aeolian Harp
When wind blows through wires or around the eves of a house mournful sounding tones are produced. This happens because alternating eddies are shed from the obstacle in the flow. These eddied are called Karman vortices after the scientist who studied them.

The frequency may be estimated from the following relation:

F= \frac{St U}{D},

Where:  St is a dimensionless number called the Strouhal Number (for a cylinder the value is .2),

U is the flow speed, and
D is the diameter

7. Door Squeaks and Chalk Squawks

Although it may seem a bit strange to think of a squeaking door or chalk sounds as musical instruments, they could easily be a valuable addition in playing some pieces of music.

Assuming that the frequency generated is a function of the following parameters

F is a function of :  U the speed of motion
\mu the molecular viscosity
D the diameter of the hinge
M The mass of the door or force on the chalk
Lr The roughness scale of the hinge or chalkboard imperfections
\rho the density of the medium

You may expect F is proportional to \frac{U D}{Lr^2} for the door, and F is proportional to \frac{U}{Lr} for the chalk
This analysis may also apply to the analysis of the sounds made by running your finger along the top of a wine glass except with the addition of the resonance of the glass itself.

8. Longitudinal Vibration of a Bar

The wave propagation speed, $C_b$, is

$$C_b = \left(\frac{Y}{\rho}\right)^{1/2}$$

Where $Y$ is the stiffness modulus, and
$\rho$ is the material density
$L$ is the length

Clamped/Free bar: $F = (2N-1)C_b/4L$

Bar fixed on both ends: $F = NC_b/2L$

9. Membranes

For membranes the tension is more important than the stiffness as a restoring force.

The wave speed $C_m = (T/\sigma)^{1/2}$, where $\sigma$ is the mass per unit area.

A kettledrum would also include the compressibility of the closed volume.

For a circular membrane:

$$F = 2.4(T/\sigma)^{1/2}/(2\pi a)$$

For a rectangular membrane:

$$F = C_m/2[\{Nx/Lx\}^2 + \{Nz/Lz\}^2]^{1/2}$$
10. Thin Plates

For thin plates stiffness is more important than tension as a restoring force.

For a circular thin plate:
F = .47 \( \frac{t}{a^2} \left[ \frac{Y}{(\rho \{1-\sigma^2\})^{1/2}} \right] \)

t is the thin plate thickness

11. Vibrations of rocks

A frequency may be estimated for the sound generated by striking two rocks together. Find two approximately spherical rocks to test this.

F = \( \frac{E}{\rho} \)/L, where L is the characteristic length (the radius in this case), E is the modulus of elasticity, and \( \rho \) is the density. This relation says that bigger rocks will produce lower frequencies.

12. Balloon Sounds

There are a rich variety of sounds that can be made using a balloon. It is tempting to use one to create some sort of bagpipe type of instrument. In this case one could allow air to escape slowly through a constricted neck. How is the frequency related to the pressure in the balloon and the size of the opening? Rubbing a balloon can also produce wonderfully annoying sounds.

The Process

1. Make calculations to guide your designs
2. Build the prototypes based upon these calculations
3. Test the prototypes
4. Analyze the results and make any necessary changes to bring your instruments to meet specs.
5. Document your procedures, designs, and results, comparing theory with experiment in a final report
6. Present a power point presentation on the exercise
7. Participate in the orchestra finale (Hocketing)

Appendix A

From H. Helmholtz On the Sensations of Tone 1877

<table>
<thead>
<tr>
<th>Notes</th>
<th>Contra Octave C to B, 16 foot</th>
<th>Great Octave C to B 8 foot</th>
<th>Uncotted Octave c to b 2 foot</th>
<th>Once-accented Octave c' to b' 4 foot</th>
<th>Twice-accented Octave c'' to b'' 1 foot</th>
<th>Thrice-accented Octave c''' to b''' ½ foot</th>
<th>Four-times accented Octave c'''' to b'''' ¼ foot</th>
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<tbody>
<tr>
<td>C</td>
<td>33</td>
<td>66</td>
<td>132</td>
<td>264</td>
<td>528</td>
<td>1056</td>
<td>2112</td>
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<tr>
<td>D</td>
<td>37 1/2</td>
<td>74 1/2</td>
<td>148 1/2</td>
<td>297</td>
<td>594</td>
<td>1188</td>
<td>2376</td>
</tr>
<tr>
<td>E</td>
<td>41 1/2</td>
<td>82 1/2</td>
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<td>330</td>
<td>660</td>
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<td>440</td>
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<td>1980</td>
<td>3960 *</td>
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<tr>
<td>B</td>
<td>61 3/4</td>
<td>123 3/4</td>
<td>247 3/4</td>
<td>495</td>
<td>990</td>
<td></td>
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</table>

The lowest tone on orchestral instruments is the E, of the double bass, making 41 1/4 vibrations in a second.† Modern pianofortes and organs usually go down to C, ¶