The Bernoulli distribution

Consider a random variable $x$ that can take one of two values: zero and one, such that $P(1) = p$ and $P(0) = (1 - p)$ where $0 \leq p \leq 1$. Is $x$ a continuous or a discrete random variable? Discrete. The density function for $x$ is

$$P(x) = f_X(x) = f_X(x; p) = \begin{cases} p^x(1 - p)^{1-x} & \text{for } x = 0 \text{ or } 1 \\ 0 & \text{otherwise} \end{cases}$$

or

$$
\begin{align*}
  f(1) &= p \\
  f(0) &= 1 - p \\
  f(x) &= 0 \text{ if } x \text{ does not equal } 0 \text{ or } 1
\end{align*}
$$

Convince me that this is a legitimate density function. Since $0 < p < 1$, $p$ and $(1 - p)$ are both positive the function is never negative. In addition,

$$f_X(0) + f_X(1) = (1 - p) + p = 1$$

Does this density function have a name? It is the Bernoulli distribution. What is $E[x]$?

$$E[x] = \sum_{x=0}^{1} x f_X(x) = 0(1 - p) + 1(p) = p$$

What is $var[x]$?
\[ \text{var}[x] = E[(x - E[x])^2] = \sum_{x=0}^{1}(x - E[x])^2f_X(x) \]

\[ = \sum_{x=0}^{1}(x - p)^2f_X(x) = (0 - p)^2(1 - p) + (1 - p)^2p \]

\[ = p^2(1 - p) + (1 - 2p + p^2)p = p^2 - p^3 + p - 2p^2 + p^3 \]

\[ p(1 - p) \]

Two examples of a random variable that would have a Bernoulli distribution: which side of a fair coin results from flipping a coin \((p = 0.5)\), whether a queen is drawn when a card is randomly drawn from a deck \((p = \frac{4}{52})\).

Note that another way to express the Bernoulli distribution is

\[
P(x) = f_X(x) = f_X(x; p) = p^x(1 - p)^{1-x} \text{ for } x = 0 \text{ or } 1
\]

\[ 0 \text{ otherwise} \]