III. Economic Curvature and VI. Economic Applications of Duality Theory

Some Additional Review questions.

1. Write a two or three page paper summarizing a lecture you might give in an undergraduate intermediate micro theory course on the duality between the direct utility function and the indirect utility function.

2. Consider the following expenditure function:

\[ E(e(U, P), P^{1/2}) = \sum_{i=1}^{3} b_{ij} P_{i}^{1/2} P_{j}^{1/2} U \]

where \( b_{ij} = b_{ji} \).

Derive the system of Marshallian demand equations. Derive the system of Hicksian demand functions.

3. a) What is a profit function?
b) What are its properties?
c) What is its usefulness?
d) Give me an example of a profit function and then;
e) Use it to derive some if...then statements about the behavior of the firm.

4. Demonstrate that the production function \( X = L^{3/5}K^{5} \), is strictly quasi-concave in terms of \( L \) and \( K \). Hint: think about the shape of the isoquants for this production function and whether this production function is increasing in its arguments. As part of your answer define quasi-concavity of the production function in terms of \( L \) and \( K \).

5. Why must the cost function be homogeneous of degree one in input prices?

6. Joe Snerd assumed the following functional form for the cost function:

\[ c(x, w, r) = (\alpha w + \beta r)x^{\gamma} \]

What is the production function that is the dual of this cost function? Explain how you figured it out.

7. Give me an intuitive (non-mathematical) economic explanation of why the cost function is concave in input prices.
8. Choose a particular functional form for a cost function. What predictions about the conditional demand for the inputs can be derived from the assumed cost function? Have you developed a theory? Specify a production function that will imply the same predictions concerning the conditional demands for the inputs. What do I want you to learn from this question? (Hint: I wanted you to learn two things.)

9. What is a cost function? What properties, if any, would one expect all feasible cost functions to have? Why? Give a specific example of a cost function. Show that your cost function possess the properties you outlined above. Does your definition of a cost function allow for multi-product firms?

10. Assume the production function \( y = f(L,K) = aL + ßK \) where \( a, ß > 0 \). Assume the exogenous input prices \( w \) and \( r \). Derive the firm's cost function. In the process of deriving the cost function, first identify the firm's conditional demand functions for labor and capital.