Review questions Economics 4808

Set 2
revised March 20, 2012

1. Wilbur, the evangelist, is in the business of saving souls. If he can get you into his church on Sunday morning (church capacity 46), your soul will be saved and you will donate $47 dollars to his ministry. (Assume that once saved, one transfers to another church.) After much research, Wilbur has discovered that he can entice new recruits into his church with the promise of money. Specifically

\[ R = 14 + 0.5P^2 \]

where \( R \) is the number of people who will attend Wilbur’s service on Sunday morning and \( P \) is the amount he promises to pay each of them to attend (he promises everyone the same amount). Further assume Wilbur likes money in addition to saving souls but everything else constant, would rather save more rather than less souls.

Describe, in general terms, the conditions that would have to be met for Wilbur to be in equilibrium from week to week. Now determine the specifics of that equilibrium. Note that \( P \) can be either positive (he pays them to attend) or negative (they pay him to attend).

**answer:** What is Wilbur’s primary objective? It is to max the number of souls saved (the first sentence says “in business to save souls”. To do this he needs to fill the church every week (get 46 people to come. Given this he want to spend the least amount of money to accomplish this. Wilbur promises to pay $8 to attend his church (solve \( 46 = 14 + 0.5P^2 \). Solution is: \( \{P = 8.0, P = -8.0\} \). If Wilbur paid $8 his profits would be -$1794. However, Wilbur could also fill the church by paying any amount greater than or equal to $8. If he did, not everyone would get in who wanted to get in. Interestingly, Wilbur could also fill the church by charging $8 or any greater amount to attend. So, the real answer, is Wilbur will charge infinity to attend his church.

2. Define the concept of “equilibrium” within the confines of an economic model. Provide an example of an economic equilibrium.

**Answer:** Economics models contain variables (things that vary). The variables that one is trying to explain within the confines of the model are call endogenous variables. Those variables who values are determined outside of the model are call exogenous variables. If, for given values (or given paths) of the exogenous variables, there is no tendency for the values of the endogenous variables to change (or change paths) the model is in equilibrium.

In a model to explain equilibrium quantities and prices for some commodities, the equilibrium condition is supply equals demand for all of the
commodities. In a model to explain the behavior of a consumer facing constraints, the individual is in equilibrium if she is doing the best she can given her constraints. In which case she does not want to change her behavior. A competitive firm that is maximizing its profits is in equilibrium because he cannot do better, so has no incentive to change its behavior.

Note that saying that "supply = demand" is an example of an equilibrium, not a definition. Lots of economic equilibriums do not entail an equality between supply and demand. For example, the consumer is in equilibrium when she is doing the best she can given her constraints.

An example is not a definition.

3. Define, in a few sentences, the concept of static equilibrium within the confines of an economic model.

**Answer:** An economic model is in static equilibrium if, for given values of the exogenous variables, the endogenous variables have no tendency to change values. Note that one can’t use the word equilibrium to define equilibrium.

Consider Model 1 from the class notes

Definitions:

\[ x^d_1 \equiv \text{the quantity demanded of good 1} \]
\[ x^s_1 \equiv \text{the quantity supplied of good 1} \]
\[ p_i \equiv \text{the price of good } i \]

Assumptions:

\[ p_2, \text{ and } p_3 \text{ are exogenous} \]
\[ x^d_1 = x^d_1(p_1, p_2, p_3) \]
\[ x^s_1 = x^s_1(p_1, p_2, p_3) \]
\[ x^d_1 = x^s_1 \]

Make the assumptions in this model specific enough so that you can solve for the equilibrium values of the endogenous variables and then derive some comparative static predictions. Then solve for the equilibrium values and derive some comparative static predictions (describe in words what they say). As part of your answer explain what you have assumed about the relationships between the three goods (substitutes, compliments, etc.).

4. Define, in a few sentences, the concept of a comparative static prediction

**Answer:** A comparative static prediction is a prediction about what happens to the equilibrium value of an endogenous variable in a model when the value of one of the exogenous variables in the model changes in value (the value of all of the other exogenous variables remaining constant).

5. Determine the equilibrium price and quantity if

\[ x^d = 14 - p^2 \]
\[ x^s = -13 + 2p^2 \]
\[ x^d = x^s \]

**Answer:** Consider the more general model
\[ x^d = a - bp^2 \]
\[ x^s = ap^2 - d \]
\[ x^d = x^s \]
\[ p^e = \pm\left(\frac{a + d}{b + c}\right)^{1/2} \]

So, there are two possible solutions. Do they both make sense? Solving for \( x^e \)
\[ x^e = a - b\left(\frac{a + d}{b + c}\right)^{1/2} = a - b\left(\frac{a + d}{b + c}\right)^{1/2} \]

In the specific example, \( a = 14 \), \( b = 1 \), \( c = 2 \), and \( d = 13 \). Plugging these in \( p^e = \pm3 \) and \( x^e = 5 \)

6. Determine the equilibrium price and quantity if
\[ x^d = 5 - 3p \]
\[ x^s = 12 + 4p \]
\[ x^d = x^s \]

Now graph the demand function and supply function with $ on the vertical axis. Identify the slope and vertical intercept for each function. Identify the equilibrium price and quantity on the graph

**Answer:** Substitute the demand and supply function into the equilibrium condition
\[ 5 - 3p = 12 = 4p \]

Solve for \( p^e = -1 \). Substitute this into either the supply or demand function to determine that \( q^e = 8 \). Does this model make sense? Not if \( x \) is a desirable commodity. How about if \( x \) is garbage?

7. Determine the equilibrium prices and quantities if
\[ x^d_1 = 5 - 3p_1 - p_2 \]
\[ x^s_1 = 1 + 4p_1 \]
\[ x^d_1 = x^s_1 \]

and

3
\[ x_d^2 = 15 - 3p_2 \]
\[ x_s^2 = 2 + 4p_2 + p_1 \]
\[ x_d^2 = x_s^2 \]

answer: In equilibrium
\[ 5 - 3p_1 - p_2 - 1 - 4p_1 = 0 \]
\[ 15 - 3p_2 - 2 - 4p_2 - p_1 = 0 \]

Simplifying
\[ 4 - 7p_1 - p_2 = 0 \]
\[ 13 - 7p_2 - p_1 = 0 \]

One way to proceed: Multiply the first equation through by $-7$ to get $-28 + 49p_1 + 7p_2 = 0$ then add this version of the first equation to the second ($13 - 7p_2 - p_1 = 0$) to get $-15 + 48p_1 = 0$, Solution is: \( p_1 = \frac{5}{16} \). So \( p_1^* = \frac{5}{16} \) = 0.3125.

Plugging this into \( 4 - 7p_1 - p_2 = 0 \) and solving for \( p_2 \) one gets \( 4 - 7(\frac{5}{16}) - p_2 = 0 \), Solution is: \( p_2^* = \frac{29}{16} \) = 1.8125. Checking, the other way, to make sure I got this right $13 - 7p_2 - \frac{5}{16} = 0$, Solution is: \( p_2^* = \frac{29}{16} \), so I did.

Now determine the equilibrium quantities of \( x_1 \) and \( x_2 \), \( x_1^e = 1 + 4(\frac{5}{16}) = \frac{9}{4} = 2.25 \), and \( x_2^e = 15 - 3(\frac{29}{16}) = \frac{153}{16} = 9.5625 \).

To do one final check on the math, let’s calculate the equilibrium quantities using the other functions. \( x_1^e = 5 - 3p_1^* - p_2^* = 5 - 3(\frac{5}{16}) - (\frac{29}{16}) = \frac{9}{4} \) and \( x_2^e = 2 + 4(\frac{29}{16}) + (\frac{5}{16}) = \frac{153}{16} \). So, everything seems to check. Visualizing what is going on

Demand and supply for \( x_2 \) given that \( p_1^* = \frac{5}{16} \) is
Demand and supply for $x_1$ given that $p_2 = \frac{29}{16}$

Note that one group solved the problem using matrix algebra (not something I will ask you to do). Consider our system of two equations

$$4 - 7p_1 - p_2 = 0$$
and 
\[ 13 - 7p_2 - p_1 = 0 \]

It can be rewritten as 
\[ 7p_1 + p_2 = 4 \]
and 
\[ p_1 + 7p_2 = 13 \]
which can be written in the matrix form \((AP = B)\) as 
\[
\begin{pmatrix} 
7 & 1 \\
1 & 7 
\end{pmatrix}
\begin{pmatrix} 
p_1 \\
p_2 
\end{pmatrix} =
\begin{pmatrix} 
4 \\
13 
\end{pmatrix}
\]
where \(A = \begin{pmatrix} 
7 & 1 \\
1 & 7 
\end{pmatrix}, \ P = \begin{pmatrix} 
p_1 \\
p_2 
\end{pmatrix}\) and \(B = \begin{pmatrix} 
4 \\
13 
\end{pmatrix}\). Solving for \(P\), \(P = A^{-1}B\). It is possible to show that in this case 
\[
A^{-1} = \begin{pmatrix} 
\frac{7}{48} & -\frac{1}{48} \\
-\frac{1}{48} & \frac{7}{48} 
\end{pmatrix} = \begin{pmatrix} 
\frac{7}{48} & -1 \\
-1 & \frac{7}{48} 
\end{pmatrix}.
\]

So, 
\[
\begin{pmatrix} 
\frac{7}{48} & -1 \\
-1 & \frac{7}{48} 
\end{pmatrix}
\begin{pmatrix} 
4 \\
13 
\end{pmatrix} = \begin{pmatrix} 
\frac{5}{16} \\
\frac{5}{16} 
\end{pmatrix}
\]

8. Determine the price and quantity if

\[ x^d = a - bp \]
\[ x^s = -c + dp \]
\[ x^d = x^s \]

\[ a, b, c, d > 0 \ (ad - bc) > 0 \]

Now graph the demand function and supply function with $ on the vertical axis. Identify the slope and vertical intercept for each function. Identify the equilibrium price and quantity

9. Determine the equilibrium price and quantity if

\[ x^d = 8 - p^2 \]
\[ x^s = p^2 - 2 \]
\[ x^d = x^s \]

Assume the equilibrium price is positive.

**Answer:** In equilibrium, \(8 - p^2 = p^2 - 2\), Solution is: \(-\sqrt{5}, \sqrt{5}\), so the equilibrium price is \(\sqrt{5}\). Plugging this into the demand or supply function, the equilibrium quantity is \(8 - (\sqrt{5})^2 = 3\). In more detail \(8 - p^2 = p^2 - 2\) implies \(10 - 2p^2 = 0\) implies \(5 = p^2\).
10. Assume a world of only two goods: beer ($B$) and nuts ($N$). Further assume that these two goods are typically consumed together. In addition, assume that
\[ B^d = 50 + T - P_B - P_N \]
\[ B^s = 25 \]
\[ B^s = B^d \]

$B^d \equiv$ the demand for beer
$B^s \equiv$ the supply of beer
$T \equiv$ temperature in degrees Fahrenheit
$P_B \equiv$ the price of beer
$P_S \equiv$ the price of nuts

Assume $T$ is exogenous. What are the endogenous variables in this model? What is the economic relationship between beer and nuts? Determine the equilibrium price of beer? If you were to criticize this model, what would you say. How would you change the model to address the criticism?

**Answer:** $T$ is exogenous, and $B^d$ and $P_B$ are endogenous. I base that conjecture on the fact the the model specifies an equation to explain $B^d$. It would seem that $B^s$ is exogenous, since it’s value is being set by assumption. What about $P_N$? If one assumes that it is endogenous, there are more unknowns that equations and one can only solve for one endogenous variable, $P_B$, as a function of another endogenous variable, $P_N$. So, assume for the moment $P_N$ is exogenous.

When $B^d = B^s$, which should have been explicitly stated, but was not,

\[ 50 + T - P_B - P_N = 25 \]

Solving this for $P_B$ one obtains.

\[ P_B = 25 + T - P_N \]

However, given that beer and nuts are close compliments, it does not make much sense to assume that the equilibrium price of beer can change without affecting the equilibrium price of nuts. This model needs to be generalized and include $N$ and $P_N$ as endogenous variables. This can be done by adding three equations to the model: a demand function for nuts, a supply function for nuts and $N^d = N^s$.

Note that some of you initially said that $P_B$ is exogenous, but then you wrote down an equation to explain the equilibrium demand for beer, which implies that the price of beer is endogenous.

Compare this question to the next one that includes the market for nuts.
11. This question has four parts. Assume a world of only two goods: beer \((B)\) and nuts \((M)\), were

\[
B^d = 50 + T - P_B - P_N
\]

\[
B^s = 25
\]

\[
N^d = 100 - 4P_N - P_B
\]

\[
N^s = 2P_N + R
\]

\(B^d\) \equiv the demand for beer

\(N^d\) \equiv the demand for nuts

\(B^s\) \equiv the supply of beer

\(N^s\) \equiv the supply of nuts

\(T\) \equiv temperature in degrees Fahrenheit

\(R\) \equiv rainfall in inches

Determine the equilibrium prices. What additional restrictions must be imposed on rainfall and temperature to guarantee that the equilibrium price of nuts is positive? Determine what happens to the equilibrium quantity of nuts if temperature increases by one degree. Determine what happens to the equilibrium quantity of beer if temperature increases by one degree.

**Answer:** In equilibrium supply equals demand for both goods

\[
50 + T - P_B - P_N = 25
\]

\[
100 - 4P_N - P_B = 2P_N + R
\]

There are numerous ways to solve this system of equations and two unknowns. I subtracted the first equation from the second to get one equation in one unknown

\[
50 - T - 3P_N = 2P_N + R - 25
\]

Solve to get \(P_N^e = 15 - .2(T + R)\). Determine \(P_B^e\) by substituting \(P_N^e\) into the equilibrium condition for beer

\[
P_B^e = 10 + 1.2T + .2R
\]

\(P_N^e > 0\) if \(.2(T + R) < 15 \Rightarrow (T + R) < 75.\)

To determine what happens to the equilibrium quantity of beer if temperature increases by one degree, first determine \(N^e\). There are a number of ways to do this. I substituted \(P_B^e\) and \(P_N^e\) into the demand function for nuts to obtain

\[
N^e = 30 - .4T + .6R
\]
From this one can see that if $T$ increases by 1, $N^e$ will decrease by .4. Note that $\frac{dN^e}{dT} = -4$, so you just calculated a partial derivative.

$B^e$ always is 25 because $B^s = 25$. Therefore the equilibrium quantity of beer does not change when temperature or anything else changes.

12. Modify the above model/theory by changing one or more of the assumptions. If you introduce more variables, make sure to define them. Derive the equilibrium values of the endogenous variables and derive at least one comparative static prediction from you model.

13. Assume the following simple Keynesian macroeconomic model:

$$Y = C + I^0 + G^0$$

$$C = .75 + .25Y^2$$

$$I^0 = G^0 = 0$$

Solve for the equilibrium levels of income and consumption. Which equilibrium is preferred? Which will society end up at?

Answer: $Y = .75 + .25Y^2$, Solution is: \{Y = 1.0\}, \{Y = 3.0\}. They prefer the one with $Y = C = 3$ over $Y = C = 1$. Who knows which they will end up at.

14. Assume the following simple Keynesian macroeconomic model:

$$Y = C + I^0 + G$$

$$C = 3 + .025Y^2$$

$$I^0 = G^0 = 1$$

Solve for the equilibrium levels of income and consumption.

Answer: Substituting the third condition into the first, one gets

$$Y = C + 2$$

which implies

$$C = Y - 2$$

Substituting this result into the consumption function

$$Y = 5 + .025Y^2$$

Rearranging

$$Y^2 - 40Y + 200 = 0$$

Using the quadratic formula $Y^e = (34.142, 5.857)$. Are both of these answers reasonable? Is society likely to prefer one over the other? This question demonstrates that the Keynesian equilibrium can be at a low level of income.
15. Assume the following simple Keynesian macroeconomic model:

\[ Y = C + I + G + (X^0 - M) \]

\[ C = a + b(Y - T^0) \quad a > 0 \quad 0 < b < 1 \]

\[ G = gY \quad 0 < g < 1 \]

\[ M = mY \quad 0 < m < 1 \]

\[ I = iY \quad 0 < i < 1 \]

\[ b + m + i < 1 \]

\( Y \equiv \) national income

\( C \equiv \) consumption of domestically produced goods

\( G \equiv \) government expenditures

\( I \equiv \) investment

\( X^0 \equiv \) the exogenous level of exports

\( T^0 \equiv \) the exogenous level of taxes

\( M \equiv \) consumption of imported goods

What is the economic interpretation of the parameter \( m \)? What is the economic interpretation of \((X^0 - M) > 0\)? \((X^0 - M) < 0\)? Solve for the equilibrium level of national income. Determine what would happen to the equilibrium level of imports (that is, would it increase or decrease) if the marginal propensity to invest increases by a small amount? What further restrictions might one impose on the parameters that would be sufficient for the impact you just derived to be positive?

**answer:** A trade surplus, a trade deficit. Substitute for \( C, G, M, \) and \( I \) in the equilibrium condition to get, \( Y = a + b(Y - T^0) + iY + gY + (X^0 - mY) \). Solution is: \( Y^e = \frac{a - bT^0 + X^0}{1 - b - g - i + m} \). Sufficient conditions for \( Y^0 > 0 \) are \((1 - b - g - i + m) > 0\) and \((a - bT^0 + X^0) > 0\). Now find the equilibrium level of imports by substituting the equilibrium level of income into the import function; that is, multiply equilibrium \( Y \) by \( m \) to get \( M^e = \frac{m(a - bT^0 + X^0)}{1 - b - g - i + m} \). The above conditions, including the added sufficient conditions are enough to imply \( M^0 > 0 \). If \( i \) increases, the numerator stays the same and denominator becomes a smaller positive number so \( M^0 \) increases.

16. Assume the following simple Keynesian macroeconomic model:

\[ Y = C + I + G + (X^0 - M) \]

\[ C = a + b(Y - T^0) \quad a > 0 \quad 0 < b < 1 \]

\[ G = gY \quad 0 < g < 1 \]
\[ M = M^0 - mY \quad 0 < m < 1 \]
\[ I = iY \quad 0 < i < 1 \]
\[ (a - bT^0 + X^0 - M^0) > 0 \]
\[ b + m + i + g < 1 \]

\( Y \equiv \text{national income} \)

\( C \equiv \text{consumption of domestically produced goods} \)

\( G \equiv \text{government expenditures} \)

\( I \equiv \text{investment} \)

\( X^0 \equiv \text{the exogenous level of exports} \)

\( T^0 \equiv \text{the exogenous level of taxes} \)

\( M \equiv \text{consumption of imported goods} \)

\( M^0 \equiv \text{the exogenous level of imports} \)

What is the marginal propensity to import? In this world, are imports superior goods? Solve for the equilibrium level of income. Determine what would happen to the equilibrium level of imports (i.e., would they increase or decrease and by how much) if the marginal propensity to invest increases by a small amount.

**answer:** Marginal propensity to import is how much imports change when income is increased by one unit. In this theory, it is \( m \) which is between zero and negative one. That is, imports decrease as income increases. This indicates that imports are an inferior good, not a superior good.

Solve for the equilibrium level of income. That is the level of income that equates aggregate demand and aggregate supply.

\[
Y = a + b(Y - T^0) + iY + gY + X^0 - (M^0 - mY)
\]
\[
= a + bY - bT^0 + iY + gY + X^0 - M^0 + mY
\]
\[
= a - bT^0 + (X^0 - M^0) + Y(b + i + g + m)
\]

Which implies that

\[
Y - Y(b + i + g + m) = a - bT^0 + (X^0 - M^0)
\]
\[
Y(1 - b - i - g - m) = a - bT^0 + (X^0 - M^0)
\]

so

\[
Y^e = \frac{a - bT^0 + (X^0 - M^0)}{(1 - b - i - g - m)}
\]
To determine what happens to the equilibrium level of imports when the marginal propensity to invest, \( i \), increases, one has to first determine the equilibrium level of imports. This is accomplished by substituting \( Y^e \) into the import demand function

\[
M^e = M^0 - mY^e = M^0 - m \left[ \frac{a - bT^0 + (X^0 - M^0)}{(1 - b - i - g - m)} \right]
\]

What happens to this when \( i \) increases?

\( i \uparrow, 0 < i < 1 \Rightarrow (1 - b - i - g - m) \downarrow \Rightarrow m \left[ \frac{a - bT^0 + (X^0 - M^0)}{(1 - b - i - g - m)} \right] \uparrow \Rightarrow M^e \downarrow. \) In words, if the marginal propensity to invest increases, equilibrium increase, causing imports to decrease because in this model they are an inferior good.

17. Assume the following simple Keynesian macroeconomic model:

\[
Y = C + I + G + (X^0 - M)
\]

\[
C = a + b(Y - T^0) \quad a > 0 \quad 0 < b < 1
\]

\[
G = gY \quad 0 < g < 1
\]

\[
M = M^0 - mY \quad 0 < m < 1
\]

\[
I = iY \quad 0 < i < 1
\]

\[
(a - bT^0 + X^0 - M^0) > 0
\]

\[
b + m + i + g < 1
\]

\( Y \equiv \) national income

\( C \equiv \) consumption of domestically produced goods

\( G \equiv \) government expenditures

\( I \equiv \) investment

\( X^0 \equiv \) the exogenous level of exports

\( T^0 \equiv \) the exogenous level of taxes

\( M \equiv \) consumption of imported goods

\( M^0 \equiv \) the exogenous level of imports

What is the marginal propensity to import? In this world, are imports superior goods? Solve for the equilibrium level of income. Determine what would happen to the equilibrium level of imports (i.e., would they increase or decrease and by how much) if the marginal propensity to consume domestic goods increases by a small amount. What further restrictions might you impose on the parameters that would be sufficient for the equilibrium
level of imports to always decrease when the marginal propensity to consume domestic goods increases?

**answer:** This question is very similar to the previous one, except it is more difficult to determine what happens the equilibrium level of imports when the marginal propensity to consume out of domestic income, \(b\), increases. 

*Marginal propensity to import* is how much imports change when income is increased by one unit. In this theory, it is \(m\) which is between zero and negative one. That is, imports decrease as income increases. This indicates that imports are an inferior good, not a superior good.

Solve for the equilibrium level of income. That is the level of income that equates aggregate demand and aggregate supply.

\[
Y = a + b(Y - T^0) + iY + gY + X^0 - (M^0 - mY) \tag{1}
\]
\[
= a + bY - bT^0 + iY + gY + X^0 - M^0 + mY \tag{2}
\]
\[
= a - bT^0 + (X^0 - M^0) + Y(b + i + g + m) \tag{3}
\]

Which implies that

\[
Y - Y(b + i + g + m) = a - bT^0 + (X^0 - M^0)
\]
\[
Y(1 - b - i - g - m) = a - bT^0 + (X^0 - M^0)
\]

so

\[
Y^e = \frac{a - bT^0 + (X^0 - M^0)}{(1 - b - i - g - m)}
\]

To determine what happens to the equilibrium level of imports when the marginal propensity consume out of domestic income, \(b\), increases, one has to first determine the equilibrium level of imports. This is accomplished by substituting \(Y^e\) into the import demand function

\[
M^e = M^0 - mY^e
\]
\[
= M^0 - m \left[ \frac{a - bT^0 + (X^0 - M^0)}{(1 - b - i - g - m)} \right]
\]

What happens to this when \(b\) increases? This is difficult to tell by the "eyeball" method because \(b\) appears in both numerator and a denominator. One takes the partial derivative of \(M^e\) with respect to \(b\). This is not something, I would ask you to do at this time.
\[
\frac{\partial}{\partial b} \left[ M^0 - m \frac{a - bT^0 + (X^0 - M^0)}{(1 - b - i - g - m)} \right] = -m \frac{\partial}{\partial b} \left[ (a - bT^0 + (X^0 - M^0))(1 - b - i - g - m)^{-1} \right]
\]

So, the sign of \( \frac{\partial M^e}{\partial b} \) has the same sign as \( T^0(1 - b - i - g - m) - (a - bT^0 + (X^0 - M^0)) \). By assumption \((a - bT^0 + X^0 - M^0) > 0 \) and \( T^0(1 - b - i - g - m) > 0 \). I don’t think there are enough restrictions to prove that one is always greater than the other. However, in empirical applications I would expect the exogenous level of net expenditures to be greater than the exogenous level of taxes multiplied by the marginal propensity to spend on “other” stuff.

18. Assume the following simple Keynesian macroeconomic model:

\[
Y = C + I^0 + G
\]

\[
C = a + .75(Y - T), \quad a > 0
\]

\[
G = 10 + gY, \quad 0 < g < 1
\]

\[
T = tY, \quad 0 < t < 1
\]

\[
(.75t - g) > -.25
\]

\[
I^0 > 0
\]

\( Y \equiv \) national income

\( C \equiv \) consumption of domestically produced goods

\( G \equiv \) government expenditures

\( I^0 \equiv \) is the exogenous level of investment

\( T \equiv \) the level of taxes

Determine how much the equilibrium level of taxes changes when the exogenous level of consumption increases. Do equilibrium taxes increase or decrease when the exogenous level of consumption increases? How is the influence of the exogenous level of consumption on equilibrium taxes affected when the tax rate increases?
answer: \[ Y = a + .75(Y - (tY)) + I^0 + 10 + gY, \]
Solution for \( Y^e \) is \( a + I^0 + 10 \). This is the equilibrium level of income. It is positive: the numerator is positive because everything in it is positive by assumption. The denominator is positive because \((.75t-g) > -.25 \implies (.25+.75t-g) > 0\). So \( Y^e > 0 \).

The equilibrium level of taxes is therefore \[ T^e = tY^e = \frac{t(a+I^0+10)}{(.25+.75t-g)} > 0 \], positive because \( t > 0 \).

What happens to this when \( a \) increases? \[ \frac{\partial T^e}{\partial a} = \frac{\partial(t(a+I^0+10))/\partial a}{(t(.25+.75t-g))} = \frac{t}{(t(.25+.75t-g))} \]
In words, when the exogenous level of consumption increases by a marginal amount, equilibrium tax revenues increase by \( t/(.25+.75t-g) > 0 \).

To determine at what rate it is changing take the derivative of \( \frac{\partial T^e}{\partial a} \) wrt \( t \)
\[
\frac{\partial}{\partial t} \left( \frac{\partial T^e}{\partial a} \right) = \frac{\partial}{\partial t} \left( \frac{t}{(.25+.75t-g)} \right) = \frac{1}{.75t-g + .25} - \frac{.75t}{(0.75t-g+0.25)^2}
\]
19. Assume the following simple Keynesian macroeconomic model:
\[ Y = C + I + G^0 \]
\[ C = a + bY \quad a > 0 \quad 0 < b < 1 \]
\[ I = c + dY \quad 0 < d < 1 \]
\[ b + d < 1 \]

\( Y \equiv \) national income
\( C \equiv \) consumption
\( G \equiv \) government expenditures
\( I \equiv \) investment

Solve for the equilibrium levels of income, consumption, and investment assuming \( a = 5 \), \( b = .5 \), \( c = 10 \), and \( d = .3 \), and \( G^0 = 7 \). Show all of your work.

answer: We these restrictions on the parameters and exogenous variables
\[ Y = C + I + 7 \]
\[ C = 5 + .5Y \]
\[ I = 10 + .3Y \]

Substitute the second and third equation into the first to obtain
\[ Y = 5 + .5Y + 10 + .3Y + 7 \]
Solve for $Y$ to obtain $Y^e = 110$. Plug this into the consumption function to obtain $C^e = 60$. Plug it into the investment function to obtain $L^e = 42$. Checking $110 = 60 + 43 + 7$.

20. Write a brief essay about what you have learned about theories and models from answering questions 13 - 19.

21. Assume the following simple Keynesian macroeconomic model:

\[
Y = C + I^0 + G^0
\]

\[
C = a + bY^2
\]

$I^0 = 20, G^0 = 25, a = 5$ and $b = .5$

$Y \equiv$ national income
$C \equiv$ consumption
$G \equiv$ government expenditures
$I \equiv$ investment

Solve for the equilibrium levels of income. What’s going on?

**Answer:** Solving

\[
.5Y^2 - Y + 50 = 0
\]

Applying the quadratic formula, $Y^e = 1\pm(1-100)^{1/2}/1 = 1\pm(99)^{1/2}(-1)^{1/2} = 1\pm9.94i$. The two solutions are imaginary numbers. This is a goofy macro model.

22. Your task is to build a simple theory of supply and demand for product $x$. I want your theory to predict the equilibrium price and quantity of product $x$. I want your theory to have two exogenous variables. Derive the equilibrium price and quantity from you definitions and assumptions. In addition derive three comparative static predictions from your theory.

23. Assume a world of just two goods, $x_1$ and $x_2$. Further assume the individual can rank all bundles of these two goods and this ranking can be represented by the utility function

\[
u = u(x_1, x_2) = a + x_1^2x_2
\]

Convince the reader that this preference ordering can also be represented by the utility function

\[
U = U(x_1, x_2) = 2\log x_1 + \log x_2
\]

24. Describe consumer theory in a nutshell. In your description do not use any terms that include the word *utility*. 
25. Define the concept of a utility function and its purpose in life. For the purposes of the question you can assume a world with only two goods, \( x_1 \) and \( x_2 \).

26. Consider the U.S. market economy. We often discuss its equilibrium properties. In introductory and intermediate microeconomics we learn that, under certain conditions, the competitive market equilibrium allocation will be efficient. All this assumes that such an equilibrium can exist. Is it obvious that it can? How can one show that an equilibrium exists?

27. What is the existence problem in equilibrium models. How can one be assured that a model does have an equilibrium. Make up a simple model of supply and demand that does not have an equilibrium price or quantity.

28. Assume the following simple Keynesian macroeconomic model:

\[
Y = C + I^0 + G \\
C = a + b(Y - T^0), \quad a > 0, \quad 0 < b < 1 \\
G = gY, \quad 0 < g < 1 \\
b + g < 1
\]

\( Y \equiv \text{national income} \)

\( C \equiv \text{consumption of domestically produced goods} \)

\( G \equiv \text{government expenditures} \)

\( I^0 \equiv \text{is the exogenous level of investment} \)

\( T^0 \equiv \text{exogenous level of taxes} \)

Solve for the equilibrium level of income. Show all your work. Determine the equilibrium level of consumption. Assuming \( Y^e > 0 \) what does the theory predict will happen to the equilibrium level of income if the government’s marginal propensity to spend increases? What happens to the equilibrium level of consumption if the exogenous level of taxes increases. Answer as specifically as possible and show all of your work.

**Answer:**

\[
Y^e = \frac{a - bI^0 + I^0}{1 - b - g}
\]

How would you interpret the numerator? the denominator? Substitute \( Y^e \) into the consumption function to determine the equilibrium level of consumption

\[
C^e = \frac{bI^0 + (1 - g)(a - bT^0)}{(1 - b - g)}
\]

\( g \) is the marginal propensity to consume on the part of the government. Examining \( Y^e \) one see that if \( g \) increases the denominator becomes smaller,
which causes the equilibrium level of income to increase. [As an aside, determining what happens to the equilibrium level of consumption when \( g \) increases is more difficult because \( g \) appears in both the denominator and numerator and the effects work in opposite directions. To determine the answer we would need to take the partial derivative of \( C^e \) with respect to \( g \) and then see if it always of the same sign.] Now consider how the equilibrium level of consumption when \( T^0 \) increases. Note that one can write

\[
C^e = \frac{bI^o + (1 - g)(a - bT^0)}{(1 - b - g)} - \frac{bI^o + (1 - g)a - (1 - g)bT^0}{(1 - b - g)} = \frac{bI^o + (1 - g)a + b(g - 1)bT^0}{(1 - b - g)}
\]

Examining the equation for the equilibrium of consumption, we see that if \( T^0 \) increases \( C^e \) decreases. How much?

\[
\frac{\partial C^e}{\partial T^0} = \frac{b(g - 1)}{(1 - b - g)} = \frac{-((1 - g)b}{(1 - b - g)} < 0
\]

because the denominator is positive by assumption and numerator is negative because \( b \) and \( g \) are positive and \( g < 1 \). You need to indicate the sign of the effect.

29. Consider Wilber. Wilber has preferences over bundles of goods. Let \( X^0 \) denote the set of bundles that Wilber can currently afford. Further assume that Wilber always chooses the bundle that he likes the best from those he can afford. Call the bundle that he chooses given the constraint \( X^0 \), \( x^0 \). Now things change such that the set of bundles he can afford is \( X^1 \) where \( X^0 \subset X^1 \). Convince the reader, in words, that this change in the constraint, cannot make Wilbur worse off.

answer: Since the original set of affordable bundles is a subset of the new set of affordable bundles, after the change the individual can still afford his best bundle given \( X^0 \). That is, he can duplicate how well he did in the original case. So, in the new case he will do at least as well as in the old case.

30. (10 pts total) This question has four parts. Assume a world of only two goods: Busch beer (\( B \)) and Heinz catsup (\( H \), where

\[
B^d = 50 + R - P_B - P_H
\]

\[
B^s = 25 + .5R
\]
\[B^d = B^s\]
\[H^d = 100 + D - 4P_H - P_B\]
\[H^s = 2P_H\]
\[H^d = H^s\]

\(B^d\) ≡ the demand for beer
\(H^d\) ≡ the demand for catsup
\(B^s\) ≡ the supply of beer
\(H^s\) ≡ the supply of catsup
\(R\) ≡ exogenous number of Republicans
\(D\) ≡ exogenous number of Democrats

Determine the equilibrium prices (3 pts for each). What additional restrictions must be imposed on the number of Democrats and Republicans to guarantee that the equilibrium price of catsup is positive? (2 pts). Determine what happens to the equilibrium quantity of beer if the number of Democrats increases? (2 pts).

**Answer:** In equilibrium supply equals demand for both goods

\[50 + R - P_H - P_H = 25 + .5R\]
\[100 + D - 4P_H - P_B = 2P_H\]

There are numerous ways to solve this system of equations. One way is to solve the first equation for \(P_B\) to get \(P_B = 0.5R - P_H + 25\) and then plug this into the second equation to get \(100 + D - 4P_H - (0.5R - P_H + 25) = 2P_H\) and solve for \(P_H\). Solution is: \(P_H = .2D - .1R + 15\), the equilibrium price of Heinz catsup. \(P_H > 0\) if \(2D - .1R + 15 > 0\) which implies \(.2D - 15 > .1R\) which implies \(2D - 150 > R\).

Now determine the equilibrium price of beer. Determine \(P_B\) by substituting \(P_H\) into the equilibrium condition for beer \(50 + R - P_B - (.2D - 0.1R + 15) = 25 + .5R\). Solution is: \(P_B = .6R - .2D + 10\). Or you could have substituted \(P_H\) into the second equilibrium condition.

Nothing happens to the equilibrium quantity of beer if the number of Democrats increases, the equilibrium quantity of beer is \(25 + .5R\).

Note that \(P_B = 0.5R - P_H + 25\) is not a solution. It is one endogenous variable as a function of another. This was a common mistake, and inexcusable.

Some of you answered a different question. I did not ask what happens to \(P_B\) when the number of Democrats increases. You need to read the questions carefully and answer what is asked.
31. (10 points) Assume the following simple Keynesian macroeconomic model:

\[ Y = C + I + G \]

\[ C = a + b(Y - T^0) \quad a > 0, \quad 0 < b < 1 \]

\[ G = G^0 - gY \quad 0 < g < 1, \quad G^0 > 0 \]

\[ I = iY \quad 0 < i < 1 \]

\[ a + G^0 > bT^0 \]

\[ b + i - g < 1 \]

\( Y \equiv \text{national income} \)

\( C \equiv \text{national consumption} \)

\( G \equiv \text{government expenditures} \)

\( I \equiv \text{investment} \)

\( T^0 \equiv \text{the exogenous level of taxes} \)

Does the equilibrium level of income increase or decrease when \( g \) increases?

Does the equilibrium level of investment increase or decrease when \( g \) increases? Describe in words the meaning of \( g \) and what it means when \( g \) increases in absolute value. Start by solving for the equilibrium level of income. Show all of your work and explain, in words, each of your steps. Make sure to keep clear the distinction between \( g \) and \( -g \). Note that when \( g \) increases, \( -g \) becomes a larger negative number. Make sure to answer all parts of this question.

**answer:** \(-g\) is the marginal propensity to spend on the part of the government; that is, if income increases by \$1 government expenditures will decrease by \( g \); that is, \( g \) is how much government expenditures decreases when \( Y \) increases by one unit. E.g. if \( g = .3 \), government expenditures will decrease by 30 cents every time national income increase by one dollar. (1 pt for knowing what \( g \) means)

I will start by determining the equilibrium level of income (4 pts)-this is a necessary part. Set aggregate demand equal to aggregate supply

\[ Y = a + b(Y - T^0) + iY + G^0 - gY \quad (4) \]

\[ = a + bY - bT^0 + iY + G^0 - gY \quad (5) \]

\[ = a - bT^0 + G^0 + Y(b + i - g) \quad (6) \]

Which implies that

\[ Y - Y(b + i - g) = a - bT^0 + G^0 \]

\[ Y(1 - b - i + g) = a - bT^0 + G^0 \]
so

\[ Y^e = \frac{a - bT^0 + G^0}{(1 - (b + i - g))} = \frac{a - bT^0 + G^0}{(g + 1 - (b + i))} \]

By assumption the numerator is positive \((a - bT^0 + G^0) > 0\), so is the denominator. So, equilibrium income is positive.

Examining the function for equilibrium income, one see that if \(g\) increases,
equilibrium income decreases. That is, if the marginal propensity to con-
sume on the part of the government becomes a larger negative number,
equilibrium income will decline. (1 pt for saying equilibrium level of in-
come decreases and 1 point for explaining why)

Now consider what happens to the equilibrium level of investment. The
easiest way to determine this is to note that \(I = iY\) \(0 < i < 1\), so
\(I^e = iY^e\). That is, equilibrium investment increases when equilibrium
income increases, so decreases when equilibrium income decreases. So an
increase in \(g\) will cause equilibrium investment to decline. Being more
specific,

\[
I = +iY^e = i \left[ \frac{a - bT^0 + G^0}{(g + 1 - (b + i))} \right]
\]

What happens to this when \(g\) increases? It decreases but by less than
equilibrium income decreased. That is, \(Y^e\) decreases and \(I^e\) is a fraction
of \(Y^e\) so it declines less. (1 pt for determining equil level of investment,
1 point for figuring out it decreases when \(g\) increases, and 1 point for
explaining why).

**Further comments:** You cannot simply say that something will go up,
or go down. You need to explain why?