1. Define, in a few sentences, the concept of an economic theory.

2. Define a thing or action in words. Refer to this thing as action $A$. Then define a condition that is necessary but not sufficient for $A$ to occur; explain why. Then, define a condition that is sufficient but not necessary for $A$ to occur; explain why. Now define a condition that is both necessary and sufficient for $A$ to occur; explain why.

**Some general comments:** you needed to begin by "defining" the action or thing you would treat as $A$. "Define" means you need to give it a name and then associate a definition with that name. E.g. "$A$ is a duck and I am defining a duck as ....." Just saying $A$ is a duck is not enough. I can’t tell whether your following answers are true until I know your definition of a duck. Further note that the set of conditions that are necc and suff for something to be $A$ is the definition of $A$. I basically asked you to give me the definition of $A$ twice. Let me again stress, whether something is necc but not suff for $A$, or suff not not necc for $A$, completely depends on how $A$ is defined.

3. (5 pts total) What is the difference between a definition and an example. That is, define what is meant by a definition, then what is meant by an example, then point out the difference between the two. (3 pts max). After you have answered to here, choose an economic function that we have studied; then provide a definition and an example. (2 pts max).

**Answer:** A definition of $A$ is a list of conditions that are necessary and sufficient for $A$. That is $A \Leftrightarrow$ (necc and suff conditions for $A$).

Alternatively, $B$ is an example of $A$ if $B \Rightarrow A$. Or said in terms of sets, if $B$ is an example of $A$ it is a subset of $A$, $B \subset A$

Another way of expressing the definition $A$ is the list of all of the conditions that are necessary for $A$.

Consider, for example, assuming a world of two goods, $x_1$ and $x_2$, an individual’s utility function, $u = u(x_1, x_2)$ is a mathematical function that identifies the individual’s maximum utility as a function of the amounts of $x_1$ and $x_2$ she consumes. Alternatively $u = x_1x_2^2$ is an example of a utility function. Another example would be the consumption function $C = C(Y)$, which is a mathematical function that identifies aggregate consumption as a function of aggregate income. An example would be $C = \alpha + \beta Y$.

Put simply, if one has the correct definition of some thing, one can use that definition to correctly determine whether anything you encounter is one of those things, or not one of those things.
**additional comments:** Some of you answered as if necessary and sufficient was something never discussed in class.

You were asked to choose an "economic function" and ..... A duck is not an economic function, a utility function is, so is cost function, production function, budget line, etc.

4. Consider the following two statements. (1) If you are not a fool, you are not an economist. (2) All economists are fools. What is the relationship between these two statements. Demonstrate your answer with a Venn diagram.

**answer:** The two statements are equivalent. One is of the form $A \Rightarrow B$; the other is $\neg B \Rightarrow \neg A$, where $A \equiv$ (being an economist) and $B \equiv$ (being a fool). Picture the set of all people. Then the subset that are fools. Economists are a subset of the fools. This issue reminds me of the Colridge poem. "Sir, I admit your general rule that every poet is a fool, but you, yourself, prove to show it that every fool is not a poet."

5. Consider the following two statements. (1) If you are not an economist, you are not a fool. (2) All economists are fools. Are the two statement equivalent? Yes or No (2 pts) and convince me of your answer (3 pts). What would it mean if both of these statement were true (2 pts). For your answer, let $A \equiv$ (being an economist) and $B \equiv$ (being a fool).

**answer:** The second statement is of the form $A \Rightarrow B$; the first is of the form $\neg A \Rightarrow \neg B$. Note that $\neg A \Rightarrow \neg B \iff (\neg (\neg B) \Rightarrow \neg (\neg A)) \iff (B \Rightarrow A)$. So the second statement says $A$ implies $B$ and the first statement says $B$ implies $A$: so these are two very different statement - they are not equivalent.

If $A \Rightarrow B$ and $B \Rightarrow A$ then $A \leftrightarrow B$. In words, $A$ and $B$ are the same thing. In our case economists and fools are one and the same.

As an aside, note that saying $A \Rightarrow B$ is equivalent to saying $B$ is necessary for $A$. Another equivalent statement is $B$ if $A$ (in words, $B$ happens if $A$ happens).

6. Consider the following theory. Let the universal set be all individuals. Define economist $E \equiv E$ as individuals who build weird models to explain the allocation of resources and the distribution of goods. Define fool $F \equiv F$ as individuals who pursue unobtainable goals. Then assume two things: economists are a subset of fools, and if you’re not an economist you are not a fool. Express your two assumptions in set notation (not arrows) and derive, using set theory, one prediction from your theory. Convince the reader that your prediction follows logically from your two assumptions and two definitions. If you made a mistake and your prediction does not follow from your two assumptions, how might someone demonstrate that your prediction does not follow from you two assumptions?
answer: $E \subseteq F$ and $\neg E \subseteq \neg F$. (2 pts for each assumption in set notation) My prediction is that fools and economists are one and the same; that is, all economists are fools and all fools are economists (3 pts for a legit prediction, and 2 pts for the explanation of why this prediction follows from the two assumptions). The logic of my prediction can be easily seen with a Venn diagram. $E \subseteq F$ says that economists are a subset (type of fool), the second assumption, $\neg E \subseteq \neg F$ (non-economists are a subset of non-fools) rules out the possibility non-economists being fools. So fools and economists are one and the same. One would prove my prediction incorrect if she can show that, consistent with the assumptions, there can be either a fool that is not an economist or an economist that is not a fool (1 pt how to disprove it).

I don’t think there are other predictions, but if you correctly come up with one, we will grade accordingly.

For some of you, your prediction was simply a rewording of the assumption "economists are a subset of fools."

7. Wilbur, the evangelist, is in the business of saving souls. If he can get you into his church on Sunday morning (church capacity 46), your soul will be saved and you will donate $47 dollars to his ministry. (Assume that once saved, one transfers to another church.) The unconverted are reluctant to attend. Everything else constant, they would rather watch All-Star Wrestling. But, after much research, Wilbur has discovered that he can entice new recruits into his church with the promise of money. Specifically

$$R = 14 + 0.5P^2$$

$R$ is the number of people who will attend Wilbur’s service on Sunday morning and $P$ is the amount he promises to pay each of them to attend (he promises everyone the same amount).

Determine how much Wilbur should promise to pay you when you attend his church.

Assuming Wilbur keeps his promises, how many souls will be saved each Sunday and how much profits will Wilbur make each Sunday?

answer: What is Wilbur’s objective? It is to max the number of soul’s saved (the first sentence says "in business to save souls"). Given this, is your answer correct. Most groups who answered this question by finding that the smallest amount that Wilbur would have to pay to fill the church. The answer was $8. If Wilbur paid $8, his profits would be $1794. However, Wilbur could also fill the church by paying any amount greater than or equal to $8, so the answer is not $8 but any amount greater than or equal to $8.

8. Define the term mathematical function in one or two sentences. Provide a specific example of a mathematical function. Is a circle in two-dimensional space a function? Why or why not?
9. Assume in Bunga Bunga there is blue money and green money. In explanation everything has a blue price and a green price and when you buy an item you have pay both the blue price and the green price. Assume you have 100 blue dollars and 25 green dollars. Further assume that the price of $A$ is 10 in blue money and the price of $B$ is 10 in blue money. Further assume, the price of $A$ is 3.3333333 in green money and the price of $B$ is 1.6666666 in green money. Also assume that more is always preferred to less. Is it possible that you would choose to consume 7 units of commodity $B$? Explain. Is it possible you would consume 9 units of $A$? Explain.

**answer:** The first constraint is $100 = 10A + 10B$, Solution is: $B = 10 - A$. The second constraint is $25 = 3.333333333333A + 1.6666666B$, Solution is: $B = 15.0 - 2.0A$. Graphing these, the budget set is the intersection of the two sets. The answers are yes and no.

Yes because $B = 7$ and $A = 3$ is affordable: buying this bundle will just exhaust his blue money budget; this bundle is affordable in terms of green money but will not exhaust his green money budget. This budget is on the boundary of the intersection of the two sets. We do not know which bundle he will choose but we do not know that he will choose a bundle on the boundary of the budget set. So, the bundle 3, 7 is one possibility.

No to the second question because $A = 9$ violates the green money budget constraint: the bundle is not in his budget set.

When I have asked this question in the past, some students were confused about what the assumption "more is preferred to less" means." It means
if bundle $A$ has more of some commodities than does bundle $B$ but not less of any other commodities, then bundle $A$ is preferred to bundle $B$. This is the assumption that guarantees that the individual will operate on the boundary of their budget set rather than in the interior of their budget set. Saying it a different way, "more is preferred to less" means an individual will always prefer more of a commodity if he can get more without giving up anything. Said another way, it is the "commodities are goods" assumption.

10. Describe, in words, what the following expression means

$$\{ (x, y) : x + y = 0 \}$$

Graph this relationship in $(x, y)$ space with $x$ on the horizontal axis and $y$ on the vertical axis. Numerically identify at least one point on your graph. Does this relationship identify a function. Do the same for

$$\{ (x, y) : y \leq x \}$$

**Answer:** $\{ (x, y) : x + y = 0 \}$ identifies a straight line through the origin with a slope of negative one. Yes this set does identify a function. That is,

$$\{ (x, y) : x + y = 0 \} \iff y = -x$$

In contrast, $\{ (x, y) : y \leq x \}$ does not identify $y$ as a function of $x$. In terms of a graph of the second set. It is all the points on and below a straight line through the origin with a slope of positive one.

11. $x^5/x^{-4} =$ ?

12. Consider a table: Identify a condition that is necessary but not sufficient for something to be a table. Why is it necessary but not sufficient. As part of your answer you will need to define a table.

13. Explain why Donald Duck is an example of a duck using the terminology of necessary and sufficient.

**Answer:** Being Donald Duck is sufficient to make one a duck, but it is not necessary (lots of ducks are not named Donald). Donald Duck is a subset of all ducks, so is an example of a duck.

14. What is a set? Consider the set of points $\{ x : |x| < a \}$. Denote this set using interval notation. How does it differ from the set $\{ x : |x| \leq a \}$? What is the maximum value of $x$ in $\{ x : |x| < a \}$

**Answer:** A set is a collection of elements. Both sets define intervals on the real line. The second includes the points $a$ and $-a$, the first does not. In interval notation, the first set is $(-a, a)$ and the second set is $[-a, a]$. There is no maximum value of $x$ in $(-a, a)$. 

5
15. Assume $x > y$. Is it then the case that $x + c > y + c$, where $c$ is some constant. Explain why or why not. Does $\alpha x > y \Rightarrow x > y/\alpha$. Yes or no and explain, possibly with an example.

**Answer:** The first statement is correct; $(x > y) \Rightarrow (x + c > y + c)$. In explanation, adding a constant, be it a positive or negative number, to each side does not change the direction of the inequality. Alternatively, one can easily show that the second state is not always correct with a counterexample. That is, one can show with an example that $(\alpha x > y) \not\Rightarrow (x > y/\alpha)$. E.g. if $a = -2$, $x = -3$, and $y = -5$. The issue in the second case is the sign of $a$, not the signs of $x$ and $y$, as long as $\alpha x > y$.

One student "proved" the first statement, $x > y \Rightarrow x + c > y + c$ using a proof by contradiction. That is, suppose $x > y \not\Rightarrow x + c > y + c$. If true, then it could be that $x + c < y + c \Rightarrow x + c - c < y + c - c \Rightarrow x < y$. That is, assuming the implication does not hold leads to a contradiction of the lhs. I find this proof problematic. **He seems to use what he was trying to prove to prove what he was trying to prove.**

16. Assume a world where there are only two consumption goods, $x_1$ and $x_2$. Their respective prices are $p_1$ and $p_2$. Consider Wilbur the consumer, who has income $y$. Using set notation, identify Wilbur’s budget set. Graph it. Is it a function? Is any part of it a function? If so, what do we call it?

Now consider his utility function $u(x_1, x_2)$. For a given bundle $x^0 \equiv (x^0_1, x^0_2)$, identify, using set notation, the strictly preferred set, the weakly preferred set. What we call the boundary of this weakly preferred set?

17. It is the case that $x > 2 \Rightarrow x^2 > 4$. Is it the case that $x > 2 \Leftrightarrow x^2 > 4$. Yes or no and explain.

18. Solve the following inequalities for $y$ in terms of the other variables: $3x + 4y \leq 12$ and $px + qy \leq m$.

**Answer:** Adding the same constant (be it positive, negative or zero) does not change the direction of the inequality. That is, $qy \leq m - px$, independent of the signs of $q$ and $p$. However, whether $y \leq \frac{m - px}{q}$ depends on the sign of $q$. If $q > 0$, $y \leq \frac{m - px}{q}$. If $q < 0$, the direction of the inequality reverses and $y \geq \frac{m - px}{q}$. In the first case $q = 4$, so the direction of the inequality is preserved.

19. Is $\pi$ a rational number? Why or why not? How about 3.1415? As part of your answer define the term "rational number"?

20. For what real number $x$ is each of the following expressions defined: $\frac{x - 1}{x(x + 2)}$, and $\frac{3x}{x^2 + 4x - 5}$?

21. The relationship between a temperature measured in degrees Celsius (or Centigrade) ($C$) and Fahrenheit ($F$) is given by $C = \frac{5}{9}(F - 32)$. Find $C$
when $F$ is 32. Find $F$ when $C = 100$. Find a general expression for $F$ in terms of $C$. If it is $40^\circ F$ in Boulder and $80^\circ F$ in Miami is it correct to conclude that it is feels twice as hot in Miami? Is it correct to conclude that it is twice as hot? If one bundle of goods provides 20 utils and another bundle 10 utils, can one say that the individual likes the first bundle twice as much as the second bundle? Why or why not?

**thoughts:** Plug 32 into $C = \frac{5}{9}(F - 32)$ and solve for $C$. The answer, as we all know, is 0. To answer the second question, first solve $C = \frac{5}{9}(F - 32)$ for $F$. The answer is $F = 32 + \left(\frac{9}{5}\right)C$. Plugging in 100 for $C$ one get 212, the boiling point on the Fahrenheit scale.

40 and 80 $F$, on the Celsius scale, are 4.4 and 26.7 on the Fahrenheit scale. Note that 4.4 $\times$ 2 $\neq$ 26.7. It is not correct to conclude that Miami feels twice as hot. The feeling of hotness is ordinal (c.p., higher temperatures feel hotter) but probably not cardinal—no one says stuff like "I feel twice as hot" or "it is twice as hot today as it was yesterday." That said if we think of temperature of element as a measure of how fast the atoms are bouncing around then temperature is a cardinal concept, but that is a different concept of temperature than is how it feels.

If we assume the individual has only ordinal preferences, all we can say is that the bundle that provides 20 utils is preferred to the one that provides 10 utils. Alternatively, one could assume preferences have cardinal properties, but we typically don’t do this because it adds nothing to the predictive power of consumer theory.

22. Consider the following implications and decide: (i) if the implication is true, and (ii) if the converse implication is true: $x > y^2$, assuming $y \neq 0 \Rightarrow x > 0$.

**Answer:** $y^2 > 0$, and $x > y^2$, so by transitivity $x > y^2 \Rightarrow x > 0$; that is, the first implication is true. Now consider the converse $x > y^2 \Leftrightarrow x > 0$, which is not always true. E.g. if $x = 1$ and $y = 3$

23. Assume a few things and then derive a prediction from those assumptions. Make sure your predictions following logically from you assumptions. When grading this we will search for examples to prove you are incorrect. Hopefully we won’t find any. You will need at least 2 assumptions and your example better not be Rambo.

**answer:** I can provide only some examples. Define dogs, cats, Fred, and Shirely. Assume dogs hate cats. Assume Fred is a dog. Assume Shirely is a cat. The prediction is that Fred hates Shirely. Why? Dogs hate cats by assumption and by assumption Fred is a dog and Shirely is a cat. That is, $F \Rightarrow D \Rightarrow HC \Rightarrow HS$, so by transitivity, $F \Rightarrow HS$.

Or, assume Fred is taller than Shirely. Assume Shirely is taller than George. The prediction is Fred is taller than George, by transitivity.

Another theory. Assume $x < 5$. Assume $y = x^2$. Prediction $(y > 25) \Rightarrow x < -5$. Note that the assumption that $y = x^2$ by itself $\Rightarrow$ if $(y > 25)$,
$x > 5$ or $x < 5$. One need the other assumption to eliminate $x > 5$, that is, one needs both assumptions to get the prediction that $(y > 25) \Rightarrow x < -5$.

In explanation, the prediction has to follow from 2 or more assumptions. A prediction that follows logically from one and only one assumption is just a restatement of the the assumption, so not a prediction from the theory.

When I have asked this question in the past, few individuals actually explained why their prediction follows from their assumptions.

Ideally, one should define all of one’s terms. Sometimes whether the prediction follows from the assumptions is unclear because it depends on how things are defined and there are no explicit definitions.

24. Analyze the following epitaph using logic: Those who knew him loved him. Those who loved him not, knew him not. In addition to your logical explanation, include with your answer a Venn diagram that will help the reader to visualize what is going on.

**answer:** The two statements are equivalent. In explanation, the first statement says that \{knew him\} implies \{loved him\}. That is, \{knew him\} guarantees that you \{loved him\}. If $A \equiv \{\text{knew him}\}$ and $B \equiv \{\text{loved him}\}$, $A \Rightarrow B$.

However, $\{A \Rightarrow B\} \Rightarrow \{\text{not}B \Rightarrow \text{not}A\}$, which is the second statement, so the first statement implies the second.

Does the second statement imply the first? Yes. In explanation:

Define $\tilde{A} \equiv \text{not}A$ and $\tilde{B} \equiv \text{not}B$. In which case the second statement can be written $\tilde{B} \Rightarrow \tilde{A}$. But $\{\tilde{B} \Rightarrow \tilde{A}\}$ is equivalent to $\{\text{not}A \Rightarrow \text{not}B\}$. Hence, $\tilde{A} \equiv \text{not}(\text{not}A) \equiv A$, and $\tilde{B} \equiv \text{not}(\text{not}B) \equiv B$, so the second statement is $A \Rightarrow B$. The two statements are equivalent.

To picture what is going on, draw 7 stick men. Give five of them, those who loved him happy faces. Give three of these hats to denote that they knew him, and give the last two sad faces because they did not love him.

Those that know him are a subset of those who loved him. Another way of saying the same thing is that being in the white set (those who loved him not) implies that you are not in the blue set (those who know him).

**One more thing.** The issue is not whether the two statements are true or not (maybe everybody hated the guy but he wrote his own epitath). Just think of each of the statements as an assumption and then determine the logical relationship between the two assumptions. Some individuals argued that the knowing him and loving him must be the same thing if ”knowing him implies loving him”, inferring you could not have loved him if you did not know him. While that might or might not be true in the real world, don’t bring additional assumptions to the table.

For example, if I develop a theory and in that theory I assume $C = f(Y) = a + bY$ and I ask you to derive the predictions of that theory, do not do
so making the additional assumption that \( b = .75 \), even thought you know \( b = .75 \) in the real word.

25. What is a variable? Now distinguish between exogenous and endogenous variables.

**answer:** A variable is something that varies in magnitude, such that any specific value of a variable can be represented with a number. *Exogenous* and *endogenous* are defined with the confines of a theory/model. Those variables, whose value are determined by the theory are referred to as endogenous variables. Those variables whose value are not determined within the theory are termed exogenous variables. Which variables are exogenous and endogenous is theory specific.

26. Your task is to build a simple theory subject to the following restrictions: it cannot be an economic theory, and it must predict at least two distinct things. Also make sure it is not a theory in the notes or review questions. Make sure your theory has all of the necessary parts. I would be inclined to express my assumptions mathematically.

**answer:** As per the notes. Your theory has to have three parts: definitions, assumptions, and at least one prediction (if...then statement).

Did you identify all the three parts. What if your "theory" has only one assumption. Is it a theory? Did you demonstrate (prove) that your if...then statements
Were the predictions you derived based solely on your explicit definitions and assumptions or did some other stuff creep in?
Do your predictions really follow from your assumptions and definitions. Each prediction must follow from two or more assumptions, otherwise it is not a prediction but rather just a restatement of an assumption.

27. Assume a world where there are only two consumption goods, \(x_1\) and \(x_2\). Consider Wilbur’s utility function \(u(x_1, x_2)\). For a given bundle \(x^0 = (x_1^0, x_2^0)\), identify, using set notation, the strictly preferred set, the weakly preferred set? What do we call the boundary of this weakly preferred set? You will need to figure out what I mean by the strictly and weakly preferred set.

**answer:** The strictly preferred set is the set of all bundles that are strictly preferred to the bundle \(x^0\); that is
\[
\{(x_1, x_2) : u(x_1, x_2) > u(x_1^0, x_2^0)\}
\]
The weakly preferred set is all those bundles that are either strictly preferred to \(x^0\) or not considered inferior; that is
\[
\{(x_1, x_2) : u(x_1, x_2) \geq u(x_1^0, x_2^0)\}
\]
The boundary of the weakly preferred set
\[
\{(x_1, x_2) : u(x_1, x_2) = u(x_1^0, x_2^0)\}
\]
is the indifference curve associated with the bundle \(x^0\).

28. Consider the different ways of expressing equivalent, necessary, and sufficient:

\(A \iff B \iff (A \text{ and } B \text{ are equivalent}) \iff (A \text{ iff } B) \iff (B \text{ iff } A)\)

\(A \implies B \iff (A \text{ is sufficient for } B) \iff (B \text{ if } A)\)

\(B \implies A \iff (A \text{ is necessary for } B) \iff (B \text{ only if } A)\)

Did I get all the equivalences correct? Note the difference between \(if\) and \(only if\). Now fill in the the following blanks with either \(iff\), \(if\), or \(only if\).

\[
\begin{align*}
x^2 > 0 & \quad \text{____ } x > 0 \\
x^2 < 9 & \quad \text{____ } x < 3 \\
x(x^2 + 1) = 0 & \quad \text{____ } x = 0 \\
x(x + 3) < 0 & \quad \text{____ } x > -3
\end{align*}
\]

Explain, in words, each of your four choices.
29. Try to use three different methods to prove that

\[-x^2 + 4 > 0 \implies |x| < 2\]

**Direct proof:** \(-x^2 + 4 > 0 \iff -x^2 > -4 \iff x^2 < 4 \iff |x| < 2\). Note that two things have been proved. The initial assertion, \(-x^2 + 4 > 0 \implies |x| < 2\), and \(|x| < 2 \implies -x^2 + 4 > 0\). Note that \(x^2 < 4 \implies x < 2\) but that \(x < 2 \implies x^2 < 4\).

Now try to come up with an **indirect proof** and a proof by **contradiction**.

30. Consider the function \(y = f(x)\), where \(x\) is a scalar. What is a function and what does this function do?

**Answer:** The function \(f(x)\) associates with each value of the variable \(x\) that is in the domain of the function a unique value of the variable \(y\). Note that \(y = f(x)\) does not necessarily associate a unique \(x\) with each \(y\).

31. Define four sets: \(S_1 = \{x_1, x_2 : p_1x_1 + p_2x_2 \leq y\}\), \(S_2 = \{x_1, x_2 : p_1x_1 + p_2x_2 < y\}\), \(S_3 = \{x_1, x_2 : p_1x_1 + p_2x_2 = y\}\), and \(S_4 = \{x_1, x_2 : p_1x_1 + p_2x_2 > y\}\). What, of anything, is the relationship (in terms of subsets, unions, intersections, etc.) between these four sets. Do any of these sets define a function. What do we call these three sets if \(x_i\) is the quantity purchased of good \(i\), \(i = 1, 2\), \(p_i\) is the price of good \(i\), and \(y\) is the individual’s income. Assuming that individuals always prefer more to less, what is the most restrictive thing the can say about the bundle, \(x^* = (x_{1}^*, x_{2}^*)\), that the individual will choose. To narrow it down further we would either have to place more restrictions on the individual’s budget set, such as his mother makes him consume at least 3 units of \(x\), or know something about her preferences.

32. Is the following formula true? \(X\setminus Y = Y\setminus X\). Demonstrate your answer with a Venn diagram. Can it sometimes be true? How about \(A \cap B = A \cap C \implies B = C?\) \(A \cup B = A \cup C \implies B = C?\) How about \(A \subset B \iff A \cap B = A?\)
**Answer:** False, they are not always the same sets, but can be; False; False; and True. That $A \cap B = A \cap C \implies B = C$ is not always true can be seen with the following counter-example. Imagine two overlapping sets $A$ and $B$. Now define $C$ as $A \cap B$. In this case, the LHS of the implication holds, but $B \neq C$. That $A \cup B = A \cup C \implies B = C$ is not always true can be seen with the following counterexample. Imagine two overlapping sets $A$ and $B$. Now define set $C$ as that part of $B$ that is not in $A$. In which case, the LHS of the implication holds, but $B \neq C$. How would you demonstrate that $A \subseteq B \iff A \cap B = A$ is true? Draw $A$ as a subset of $B$.

33. Is the greatest economist among the mathematicians and the greatest mathematician among the economist one and the same person? Is the oldest economist among the mathematicians and the oldest mathematician among the economists one and the same person? Answer in terms of set theory and demonstrate your answers with Venn diagrams.

**Answer:** No and Yes. Let $E$ denote the set of all economists. Let $M$ denote the set of all mathematicians. The set of individuals that are both mathematicians and economists is $EM \equiv E \cap M$; that is, the intersection of the two sets. The greatest economist among the mathematicians, $e$, is a member of $EM$; that is $e \in EM$. The greatest mathematician among the economists, $m$, is a member of $EM$; that is $m \in EM$. But $e$ and $m$ are not necessarily the same person. For example, Sue might be the greatest economist in $EM$ and Peggy might be the the greatest mathematician in $EM$.

In contrast everyone in $EM$ is both an economist and a mathematician, so the oldest person in $EM$ is both the oldest economist and the oldest mathematician.

In terms of a Venn diagram $EM$ is the intersection of the set of economists and the set of mathematicians, so consists of all of those individuals that are both economists and mathematicians.

34. State two methods of proving the statement "$A$ is a necessary condition for $B$".

**Answer:** Show that

- $not \ A \Rightarrow not \ B$, which is the same as $B \ only if \ A$, or show that
- $B \Rightarrow A$, which is the same as $B$ is sufficient for $A$

35. What is an indirect proof of $A \Rightarrow B$. Provide an example of an indirect proof.

**answer:** This method of proof follows from

$$\{A \Rightarrow B\} \Leftrightarrow \{notB \Rightarrow notA\}$$
That is, one way to show that $A$ implies $B$ is to show that the absence of $B$ implies the absence of $A$. Example, prove that $A \equiv 5x > 0$ implies $B \equiv x > 0$. This is a very trivial example, in that it is obvious that $A$ implies $B$. But let's work through the steps.

**direct proof**

want to directly manipulate $A$ into $B$

divide $5x > 0$ by $5$ to get $x > 0$

$qed$

**indirect proof**

show $notA$ implies $notB$

assume $notB$, that is, assume $x \leq 0$

if $x \leq 0$ then $5x > 0$ is false

$qed$

**proof by contradiction**

start by assuming $A \Rightarrow B$ ($A \Rightarrow notB$)

that is, assume $5x > 0 \Rightarrow x \leq 0$

but $x \leq 0 \Rightarrow 5x \leq 0$

which contradicts $A \equiv 5x > 0$, qed.

Here is another example: Prove that $x^2 > 2 \Rightarrow x \neq 1$ using an indirect proof. In this case $A \equiv x^2 > 2$ and $B \equiv x \neq 1$. So $notB$ is $x = 1$. If $x = 1$, then $x^2 = 1 \neq 2$. So $notB \Rightarrow notA$. When I have asked this question in the past, the answers have contained no proof, indirect or otherwise.

36. How does one proceed if the intent is to prove that $A \Rightarrow B$ using proof by contradiction.

**answer:** One proceeds by assuming that $A \Rightarrow B$, and then showing that this contradicts $A$. In the example in the book $A \equiv -x^2 + 5x - 4 > 0$ and $B \equiv x > 0$. So, if $-x^2 + 5x - 4 > 0 \Rightarrow x > 0$ there must be an $x < 0$ for which $-x^2 + 5x - 4 > 0$. But, there can’t be because $x < 0 \Rightarrow -x^2 + 5x - 4 < 0$ which contradicts $A$.

37. It is the case that $x > 2 \Rightarrow x^2 > 4$. Is it the case that $x > 2 \Leftarrow x^2 > 4$. Yes or no and explain.

**Answer:** One way to disprove either statement is with a counterexample. The second statement, $x > 2 \Leftarrow x^2 > 4$ is not always true because if $x$ is $-5$, $x^2 > 4$ but $-5 \not> 2$. Now consider the first statement, $x > 2 \Rightarrow x^2 > 4$. It is true because if $x$ is greater than $2$, $x^2$ is always greater than $4$.

As an aside the way one interprets $x > 2 \Rightarrow x^2 > 4$ is if $x > 2$ then $x^2 > 4$

38. What is a utility function, what is its purpose, and how does it fulfill this purpose.

**answer:** (1) a utility function assigns a number to every possible bundle of goods. Its goal in life is to represent an individual’s preferences (ranking over the bundles of goods) in a compact way. If fulfills this goal if it assigns the same number to two bundles if the individual is indifferent between those two bundles, a larger number to the bundle the individual prefers if the individual prefers one bundle over the other. This is all I need.

Reactions to some of the things when I have asked this question in the past: Neoclassical consumer theory does not assume individual have utility
functions or get utility from stuff. It assumes just that individuals choose the bundle they like the best from the ones that can afford. Individuals don’t have utility function in their heads, they have a ranking (or at least that is what neoclassical consumer theory assumes). Individuals do not maximize utility. Utility functions have nothing to do with the constraints the world imposes on the individual. Choosing the bundle you rank the highest from those you can afford does not mean you are happy or sad.

39. Consider George the consumer. George lives on a fixed income, $m$, and only has so much time, $t$. Define $B_m$ as all those bundles of goods that George can afford to buy with his money income. Define $B_t$ as all those bundles of goods George has enough time to consume. Assume that $B_m \cap B_t$ is not empty. Further assume George has preferences such that he can compare any two bundles of goods and say which he prefers. Also assume George chooses the bundle he most prefers, $b^*$, from those he has the time and money to consume.

part 1: Is the most-preferred bundle from those he can afford to buy, $b^*_m$, necessarily the same bundle as the most-preferred bundle from among those he has time to consume, $b^*_t$? part 2: Is the chosen bundle, $b^*$, always either $b^*_m$ or $b^*_t$. part 3: Will George necessarily spend all of his time and money? part 4: Would your answer to part 2 be different if one adds the assumption that George has enough time to consume $b^*_m$?

Answer each of the first three parts of the question, 1 – 3, either Yes or No. Explain each of those answers in words and using a diagram. You can probably use the same diagram for all three questions. Now answer part 4 in words and in terms of your diagram. Your diagram is a Euler diagram in the sense that it will show the intersection of two sets. However, in this case the shape and dimensions of these two sets have meaning which you should incorporate. Think a world of two goods and what budget sets look like.

answer: No, No, No.

Draw the two sets with an non-empty intersection.

Note that no one assumed that George had enough time to consume $b^*_m$ or enough money to purchase $b^*_t$, so neither one has to be in the intersection of the two sets. Place them in the diagram such that neither is in the intersection. This proves that $b^*_m$ and $b^*_t$ are not always the same bundle. So, no is the answer to part 1.

With respect to part 2. The bundle that George consumes, $b^*$, must be in the intersection of the two sets. Since, in my example neither $b^*_m$ nor $b^*_t$ are in the intersection, in my example neither $b^*_m$ of $b^*_t$ is $b^*$. So, one of them does not have to be the consumed bundle and the answer to part 2 is no.
part 3: There is no reason his most preferred bundle among those he can “afford” has to use up all of his time and money. It could be, but this is pretty unlikely.

part 4: Yes it would change the answer because now we have added an assumption that puts $b_m^*$ in the intersection of the two sets (in the bundles he has to time and money to consume). So, now $b_m^* = b^*$. Note that it could be the case that $b_t^* \neq b^*$.

The first step in answering this question is to understand the notation (language). If you don’t understand the concepts and notation, all is lost. $B$ is a set of bundles of goods and $b$ is a particular bundle. As in, $B_m$ is the set of all bundles of goods that George can afford. $b_m$ is a bundle in $B_m$, and $b_m^*$ is a specific bundle of goods; it is the one he like the best from those he can afford to buy. Do you understand the distinction between a bundle of goods and a set of bundles of goods? $b_t^*$ is the bundle George would choose if money was not a constraint. E.g., weekend in Paris at the Ritz with lot of good food, wine and shopping. $b_m^*$ is the bundle George would choose if time was not a constraint. E.g. round the world backpacking trip, eating lots of rice. There is no reason they are the same.

Did you read question carefully?

Consider the following example. I have 10 hours of time and $100. There are two goods, $x_1$ and $x_2$. Assume good 1 is very expensive in terms of money, but takes little time to consume. Assume the opposite for good 2. Ignoring, the money constraint, I would choose a bundle with more good 1 than good 2. Ignoring the time constraint, I would choose a bundle with more good 2 than 1. Neither of these bundles is unnecessarily my most preferred bundle from the set of bundles I can afford in terms of both time and money. The diagram is courtesy of Matt (Sept 04),
Andy provided the following correct, but less informative, diagram
40. Consider the statement *One is a good economist only if one hates math.* With the different characterizations of the concepts of necessary and sufficient in mind, write down a number of equivalent statements, first in terms of $G$ and $H$, then in terms of words, where $G$ denotes a good economist and $H$ denotes those who hate math.

Answer: From the class notes and logic we know that

\[(H \text{ is necessary for } G) \Leftrightarrow \neg H \Rightarrow \neg G\]
\[(G \text{ only if } H)\]
\[(G \text{ is sufficient for } H)\]
\[(G \Rightarrow H)\]

All of these statements are equivalent. Given my definition of $G$ and $H$, *(One is a good economist only if one hates math) $\Leftrightarrow$ (G only if H)*, the third line. Therefore the statement is equivalent to all of the following statements
"Hating math is necessary condition to be a good economist, \((H \text{ is necessary for } G)\)"

"If you don’t hate math, you are not a good economist, \((\text{not}H \Rightarrow \text{not}G)\)"

"Being a good economist is sufficient to guarantee that one hates math, \((G \text{ is sufficient for } H)\)"

"Being a good economist implies that one hates math, \((G \Rightarrow H)\)"

Whether the original statement is, or is not true, is immaterial to the question. Some of you also said that \(H \Rightarrow G\) or that \(\text{not}G \Rightarrow \text{not}H\). While each of these imply the other \(((H \Rightarrow G) \iff (\text{not}G \Rightarrow \text{not}H))\), neither is correct. Hating math does not imply that one is a good economist; as one student said, "My mom hates math and she definitely is not a good economists." Or said the second way not being a good economist does not imply that you don’t hate math. One can be something other than a good economist and hate math or not hate math. So correct statements are \(H \nRightarrow G\) and \(\text{not}G \nRightarrow \text{not}H\): hating math is not enough to make you a good economist and not being a good economist is not enough to make you not hate math.

The following is my attempt at a Venn diagram of the statement \textit{One is a good economist only if one hates math}
Good economists are a subset of economists \((G \subset E)\). Good economists are also a subset of things that hate math \((G \subset H)\). By definition, economists are a subset of the universal set \((E \subset \Omega)\); everything is a subset of the universal set.

Note some other things that do not follow from the statement that \(\text{One is a good economist only if one hates math}\). For example,

Hating math does not imply that one is a good economist or even an economist \((H \not\Rightarrow E)\). That is, there are lots of math haters who are not economists.

Not hating math does not imply that one that one is an economist \((\text{not}H \not\Rightarrow E)\). That is, there are lots of things out there that do not hate math who are not economists.

Not being a good economist does not imply that you are a bad economist; you might be an OK economist \((\text{not}G \not\Rightarrow \text{bad}E)\) unless, of course, one assumes that all economists are either good or bad; that is, mediocre economists are impossible.

41. In one page or less make up and answer a question. Design your question so that to answer the question correctly one will need to understand the distinction between necessary and sufficient and be required to explain the distinction. I will grade your question in terms of its ability to teach the distinction between necessary and sufficient, and grade your answer in terms of how well it answers the question. Clarity and exposition are important, so is the ability of your question to interest and challenge the student.

42. Describe the set \(A \cup \text{not}A\).

43. Assume that neither set \(A\) nor set \(B\) are the universal set. Further assume that \(A\) and \(B\) have the following relationship, \(\text{not}A \Rightarrow B\). Does this imply that \(\text{not}B \Rightarrow A\)? Why or why not?

**answer:** The answer is yes. Let see if we can rearrange the first statement, \(\text{not}A \Rightarrow B\), so that we get the second statement, \(\text{not}B \Rightarrow A\). Start be defining \(\text{not}A\) as \(C\). In which case the first statement is \((C \Rightarrow B) \Rightarrow (C\text{ is sufficient for }B) \Rightarrow (\text{not}B \Rightarrow \text{not}C) \Rightarrow (\text{not}B \Rightarrow A)\), which is the second statement. That is the first statement impies the second.

Does the second statement imply the first? If so, the two statements are equivelant. Start with the second statement, \(\text{not}B \Rightarrow A\). Let \(D \Leftrightarrow \text{not}B\), so the second statement can be written as \((D \Rightarrow A) \Rightarrow (\text{not}A \Rightarrow \text{not}D) \Rightarrow (\text{not}A \Rightarrow B)\), which is the first statement. So each statement implies the other. The two statements are equivelant. Consider the following example, \(A\) is lean and \(B\) is fat. If not being lean implies that one is fat, then not being fat implies that one is lean. Show this with a Venn diagram.
Stating the equivalence

\((\neg A \Rightarrow B) \iff (C \Rightarrow B) \iff (C \text{ is sufficient for } B) \iff (\neg B \Rightarrow \neg C) \iff (\neg B \Rightarrow A)\),

where \(C\) is defined as \(\neg A\).

Remember that the universal set \(\Omega \equiv A \cup \neg A \equiv B \cup \neg B\): there are two ways to split everything up.

An aside, things are more complicated if, for example \(B\) is the universal set (something I assumed away at the beginning of the question. Consider an example where \(A\) is a strict subset of \(B\). in which case, \(\neg A\) is that part of \(B\) that does not include \(A\). In this case \(\neg A \Rightarrow B\). To picture this imagine \(A\) is an economist, \(B\) is fools, and everyone is a fool. In which case, \(\neg A \Rightarrow B\) (if you are not an economist you are a fool) and \(A \Rightarrow B\) (if you are an economist you are a fool) -everyone is a fool by assumption - fools are the universal set. But in this case of all fools, does \(\neg B \Rightarrow A\). No, because \(\neg B\) is the empty set. In this case, where does the string of logic I present above go wrong.

It is still the case that

\((\neg A \Rightarrow B) \iff (C \Rightarrow B) \iff (C \text{ is sufficient for } B)\)

But, it is not the case that

\((C \text{ is sufficient for } B) \iff (\neg B \Rightarrow \neg C)\)

where \(\neg C \equiv A\). In words, being in the empty set (non-fools) does not make you an economist. We have to be careful with our rules of logic when one of the sets is the universal set.

44. Make up an example for students of this class that will help them to better understand set theory, necessary and sufficient, and deriving predictions from sets of assumptions. Use a set diagram to help drive home your points. I will want to see some set theoretic notation. Begin by defining some number of sets, make some assumptions about the relationships between those sets, and then drive some predictions from those set-theoretic assumptions. Make sure to clearly explain why your predictions follow from your assumptions (you need to convince the reader). I am looking for material that I can include in the set of review question, or in the lecture notes, or on a quiz or exam.

Some criteria for evaluating the example:

- Is the prediction correct given the assumptions and definition?
- Are the assumptions and definitions consistent with one another, or do some contradict others?
- Are some of the assumptions implied by other assumptions?
- Does a "prediction" follow from one and only one assumption - if so, it is not a prediction, rather just a restatement of the assumption.
Do they understand necessary and sufficient? 
Do they understand what it means to logically deduce a prediction from a set of assumptions and definitions? 
Would a reader learn from the example?

45. Consider the statement $x(x + 3) < 0$ where $x > -3$ is a blank you must fill in with either $if$, $iff$, or "only if". Which is it, and why? 

**answer:** Start by determining that $x > -3$ is necessary for $x(x + 3) < 0$. In explanation, for $x(x+3)$ to be negative requires that one term is negative and one is positive. If $x \leq -3$ this will never be true. For example, if $x = -4$, the product will be positive. The question then is whether $x > -3$ is sufficient to accomplish $x(x + 3) < 0$. The question is no. For example, if $x = 1$, $x(x+3) \not< 0$. So, the rhs is necessary but not sufficient for the lhs, making the answer "only if" as in $x > -3$: "only if" as in required/necessary.

As in "George is having sex only if he pays $100. He will not have sex if he does not pay the $100, but the paying does guarantee that he will have sex."

On the other hand, paying if the $100 guarantees that he will sex but he might get sex without paying then "Sex if he pays"

On the other hand if he must pay $100 to have sex and will for sure have sex if he pays the $100 then its "Sex if he pays."

46. (5 points) Consider Mildred. She lives in a world of two goods. Mildred has preferences over bundles of goods such that *more is always preferred to less*. In addition, she has a budget constraint $y \geq p_1 x_1 + p_2 x_2$ where $y$ is Mildred’s income, $p_i$ is the exogenous and positive price of good $i$ and $x_i$ is the quantity of good $i$ she consumes, $i = 1, 2$. Graph the set of bundles she can afford, with $x_1$ on the vertical axis and $x_2$ on the horizontal axis. Assume all of the prices are less than $5 and that her income is at least $100. 

Now assume that the price of good 2 increases ($p_2^1 > p_2^0$). Graph the new budget set on the same diagram. Convince me that it is possible that this price increase will not make Mildred worse off. As part of your answer define *more is always preferred to less*. Give me an example of a specific utility function such that if Mildred has this utility function the increase in the price of good 2 would not make her worse off. Make sure this utility function is consistent with the assumption that more is preferred to less.

**answer:** (2 pts max for correctly drawing the budget sets, 1 pt max of identifying where the "bundle" must be, 1 pt for defining *more is always preferred to less*, and 1 pt max for a utility function). *More is always preferred to less* means that if bundle $A$ has has more of at least one good than does bundle $B$ and no less of any good in bundle $B$, $A$ is preferred to $B$. The new and old budget sets share many bundles in common but only one bundle where income is exhausted independent of which of the two
budget sets holds: the bundle where all of her income is spent on good 1. Choosing this bundle with either budget set is thus consistent with the assumption of more is preferred to less. If this bundle were her most preferred bundle with the original budget set, it could remain her most preferred bundle after $p_2$ has increased. For example, assume $y = 100$, $p_1 = 1$, $p_2^0 = 2$ and $p_2^1 = 3$.

The inner line is the boundary of the budget set after $p_2$ has increased. The only bundle in both sets is $x_1 = 100$ and $x_2 = 0$. Mildred would choose this bundle with both budget sets if, for example, her utility function were $u = 1000x_1 + x_2$. In contrast, $u = 1000x_1$ does not do it because it violates the assumption that more is preferred to less.

In terms of logic, we have demonstrated, by example, that increasing $p_2$ will not necessarily make Mildred worse off.

**further comments**: some of you had the $x_1$ intercept change when $p_2$ increased. Wow.