Econ 4211 Suggested solutions to HW2

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1 Problem 1

A person can receive 3 offers
$50,000 with probability $5/10$
$60,000 with probability $4/10$
$100,000 with probability $1/10$

Discount rate is $\beta = 0.8$

a. What is the value of unemployment benefit $(b)$ that will make the individual indifferent between accepting the lowest offer and rejecting it?

b. What is the range of benefits for which the lowest two offers will be rejected?

c. How high should be the discount rate that will make your answer to part a. $b = 0$.

The value of a lifetime offer of $50,000 per year is

$$v(50,000) = \frac{50000}{(1 - 0.8)} = 2.5 \times 10^5$$ (1)

$$v(60,000) = \frac{60000}{(1 - 0.8)} = 3.0 \times 10^5$$ (2)

$$v(100,000) = \frac{100000}{(1 - 0.8)} = 5.0 \times 10^5$$ (3)

Expected value of the offers:

$$E(w) = 10^5 \left( 2.5 \times \frac{5}{10} + 3 \times \frac{4}{10} + 5 \times \frac{1}{10} \right) = 2.95 \times 10^5$$ (4)
An alternative way to calculate the expected value of offers is the following:

\[
\frac{10000}{1-\beta} \left( 5 \times \frac{5}{10} + 6 \times \frac{4}{10} + 10 \times \frac{1}{10} \right) = \frac{59000}{1-.8} = 2.95 \times 10^5
\]

(5)

Discounted value of it is

\[
\beta E (w) = .8 \times 2.95 \times 10^5 = 2.36 \times 10^5
\]

(6)

a. Answer: \( b = v(50,000) - \beta E (w) = 2.5 \times 10^5 - 2.36 \times 10^5 = 14,000 \)

b. Answer: \( b \) should be above \( 3.0 \times 10^5 - 2.36 \times 10^5 = 64,000 \). The third offer is accepted if the compensation is below \( 5.0 \times 10^5 - 2.36 \times 10^5 = 264,000 \)

c. Answer:

The individual is indifferent between waiting for another offer and accepting the lowest one if the discounted expected value of an offer is equal to the value of the lowest offer, \( \beta E (w) = v (50,000) \). Note that both the value of the lowest offer and the expected value depend on \( \beta \).

\[
E (w) = \frac{10,000}{1-\beta} \left( 5 \times \frac{5}{10} + 6 \times \frac{4}{10} + 10 \times \frac{1}{10} \right) = \frac{59000}{1-\beta};
\]

(7)

\[
v (50,000) = \frac{50000}{(1-\beta)}.
\]

(8)

Therefore, the equality \( \beta E (w) = v (50,000) \) implies

\[
\beta \frac{59000}{1-\beta} = \frac{50000}{(1-\beta)};
\]

(9)

\[
59\beta = 50
\]

(10)

Solution is: \( \beta = \frac{50}{59} = 0.84746 \)

2 Problem 2

Assume you get an information from a credible source that in neighborhood \( X \) a high school drop-out can start a career (at the age of 15) as a drug dealer. If successful, he can retire at the age of 35 (he can accumulate \( W \) in wealth). Assume that the chance of staying alive till the age of 35 for drug dealers is \( 1/5 \). According to the same source, provided that the person
survives, there is 1/10 chance of not being imprisoned, thus enjoying the “early retirement”. Assume that this is the most attractive offer the high school drop-out is aware of by the time he leaves school.

If the “young adult” stays in school, he learns (by graduation) that there a chance (1/5) of becoming a successful lawyer and accumulate $W$ by the age of 40.

You are invited to consult a politician whose platform includes an increase in spending for public schools in neighborhood $X$. Use the information above to justify the increase.

**Answer:**

use the argument that education is a public good: it provides the students with better understanding of their possible future careers. Better schooling can decrease the drop-out rate, thus more students will learn about alternative options of earning money in neighborhood $X$, some of them may decide not to become drug-dealers as result, thus decreasing the crime rate (and increasing GDP), which will generate benefits for all the residents of the neighborhood.

### 3 Problem 4, Chapter 9, Q.3

In order to balance the pay-as-you-go system, the outlays should equal to the payments,

$$Benefits \times N_{old} = t \times N_{young} \times \text{wages}, \quad (11)$$

where $t$ is the tax rate, $N_{old}$, $N_{young}$ are the number of old (recipients of the Social security) and the number of young (those who pay) correspondingly. Thus the tax rate should be defined by the formula

$$t = \frac{N_{old}}{N_{young}} \times \frac{Benefits}{\text{wages}}. \quad (12)$$

If the ratio $\frac{Benefits}{\text{wages}}$ between the years 1990 and 2050 has to be kept constant, and the ratio of the old to young $\frac{N_{old}}{N_{young}}$ has increased from to .458, the change in the tax rate should be proportionate. In other words, denoting the tax rate in 1990 by $t_{1990}$ and the tax rate in 2050 by $t_{2050}$, we calculate that

$$\frac{t_{2050}}{t_{1990}} = \frac{.458}{.267} = 1.715 \, 4 \quad (13)$$
the new tax rate will have to be 71.5% higher.

In case the tax rate is kept constant, the benefits to wages ratio will have to fall by \( \frac{267}{458} = 0.58297 \), i.e., by 58.3%.

4 Part 4 Chapter 11, q. 3

The present value is

\[
\sum_{t=1}^{\infty} \left( \frac{1}{1+.1} \right)^t \times 25 = 25/.1 = 250
\]  

(14)

In general, if the yearly benefit is \( B \) and the interest rate is \( r \),

\[
\sum_{t=1}^{\infty} \left( \frac{1}{1+r} \right)^t \times B = \frac{1/(1+r)}{1-(1/(1+r))} \times B = \frac{B}{r},
\]

(15)

(16)

which corresponds to the claim in the textbook.

5 Part 4 Chapter 11, q. 4

a. Annual benefit of $80 with interest rate \( \rho \) yields \( 80/\rho \) by the previous question. The project will “break even” at the rate of \( \rho \) is \( 1000 = 80/\rho \). Therefore, \( \rho = .08 \).

b. If the money comes from consumer spending, we have to use the after-tax rate of interest, as it is the correct alternative cost of the funds. In this case the present value of benefits is \( 80/.05 = 1600.0 \) \( > 1000 \), so that the project’s present value is positive and it is admissible. If the money comes from investment, the corresponding alternative cost is the before-tax interest rate, in this case the present value of the project is negative, \( 80/.10 - 1000 = -200.0 \), thus it is not admissible. In the mixed case the discount rate is \( .6 \times .10 + .4 \times .05 = 0.08 \), which is exactly the internal rate of return of the project, so that the present value is zero. There is no net advantage in undertaking the project.

c. Present value of the project then is \( 80/.04 - 1000 = 1000.0 \) \( > 0 \)

d. If the inflation is fully anticipated and the market interest rates adjust accordingly, the answers are the same, as they are calculated in real terms.