Chapter 4: Open Economy models

Naturally, a very important task for the modeler is building open economy models, since trade and international investment policies are among the important policy problems facing governments.

M41.GMS Small open economy 2x2
M42.GMS Small open economy with a benchmark tariff
M43.GMS Same as M42 except with a different price normalization
M44.GMS Small open economy with a benchmark quota
M45.GMS Small open economy with a benchmark voluntary export restraint
M46.GMS Small open economy with a benchmark trade imbalance
M47.GMS An Armington formulation
Model 41  Small open economy 2x2 (two goods, two factors)

This is a very simple 2-good, 2-factor model in which the rest of the world is not explicitly modeled. Trading opportunities are summarized by simple functions which allow the economy to transform one good (an export) into another (an import). The “technology” of these functions represents world prices. We will assume that these technologies or price ratios are fixed in this model. This is commonly known as the “small-country assumption”.

We assume in the benchmark data that the country exports $X_1$ in exchange for $X_2$, but we will allow for the fact that some policy or endowment change could actually reverse the direction of trade by specifying (initially inactive) functions that transform goods in the opposite direction. Here is the initial data, in which 50 units of good $X_1$ are exchanged for 50 units of good $X_2$ at an implicit price ratio of $p_1/p_2 = 1$. 
Production Sectors

Consumer

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<tr>
<th>Markets</th>
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Technology parameters are specified in these functions that allow the modeler to change the terms of trade. These are given by:

PARAMETERS

PE2    Export price of good 2,
PM1    Import price of good 1,
PE1    Export price of good 1,
PM2    Import price of good 2,
TM2    Import tariff for good 2;
PE1 = 1;
PM2 = 1;
PE2 = 0.99;
PM1 = 1.01;

E1 and M2 production activities that are the initially active trade links. E1 stands for exports of good $X_1$. While we could specify this activity as directly transforming $X_1$ into $X_2$, in more complicated models with many goods it proves useful to define another good which we will call “foreign exchange” and whose price is denoted PFX. All trade is mediated through the “foreign exchange market”.

Thus activity E1 transforms $X_1$ into foreign exchange and M2, the import activity for good 2, transforms foreign exchange into imports of good 2. These activities are given as follows:

$\text{PROD:E1}$

\[
\begin{align*}
O: & \text{PFX} & Q: (50*PE1) \\
I: & \text{P1} & Q: 50
\end{align*}
\]
TM2 is an import tariff on good 2, which is initially set to zero. We also specify trade links in the opposite direction, which are initially inactive as noted above. At this point, we have an opportunity to make an important technical remark which may save the modeler some misery later on. Suppose that good 1 can be transformed into good 2 at a price of one, and good 2 can be transformed into good 1 at a price of 1. Then if the export of 50 units of good 1 and imports of 50 units of good 2 is an equilibrium, then so is the export of 100 units of good 1 followed by the imports of 50 units of good 2 plus 50 units of good 1.

In technical terms the model is “degenerate”, it has infinitely many solutions. In such a situation, the solver will either fail to converge, or converge to an arbitrary solution. This is why we specify the prices or “terms of trade” differently for the activities M1 and E2, so that it is never profitable to export and import the same good.

Here is the model. In the first counterfactual we impose a tariff and 5% and then a tariff of 10%. When you look at the listing files, you will see that the tariff of 10% is prohibitive, all trade ceases. The last experiment returns the tariff to zero, and improves
the terms of trade (relative prices of the export good) to 1.2.

$TITLE Model M41: Small open economy model. Two goods, two factors.

$ONTEXT

Production Sectors

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$OFFTEXT
PARAMETERS
    PE2     Export price of good 2,
    PM1     Import price of good 1,
    PE1     Export price of good 1,
    PM2     Import price of good 2,
    TM2     Import tariff for good 2;

PE1 = 1;
PM2 = 1;
PE2 = 0.99;
PM1 = 1.01;
TM2 = 0;

$ONTEXT

$MODEL:M41
$SECTORS:
X1 ! Production index for good 1
X2 ! Production index good 2
E1 ! Export level of good 1
E2 ! Export level of good 2
M1 ! Import level of good 1
M2 ! Import level of good 2
W ! Welfare index

$COMMODITIES:
P1 ! Price index for good 1
P2 ! Price index for good 2
PFX ! Read exchange rate index
PW ! Welfare price index
PL ! Wage index
PK ! Capital rental index

$CONSUMERS:
CONS ! Income level for representative agent
$PROD:X1  s:1
  O:P1     Q:150
  I:PL     Q:100
  I:PK     Q: 50

$PROD:X2  s:1
  O:P2     Q:50
  I:PL     Q:20
  I:PK     Q:30

$PROD:E1
  O:PFX    Q:(50*PE1)
  I:P1     Q:50

$PROD:M2
  O:P2     Q:50
  I:PFX    Q:(50*PM2)  A:CONS  T:TM2

$PROD:E2
  O:PFX    Q:(50*PE2)
  I:P2     Q:50
$PROD:M1
    O:P1   Q:50
    I:PFX  Q:(50*PM1)

$PROD:W   s:1
    O:PW   Q:200
    I:P1   Q:100
    I:P2   Q:100

$DEMAND:CONS
    D:PW   Q:200
    E:PL   Q:120
    E:PK   Q: 80

$OFFTEXT
$SYSINCLUDE mpsgeset M41

PW.FX = 1;
E2.L = 0;
M1.L = 0;
E1.L = 1;
M2.L = 1;

M41.ITERLIM = 0;
$INCLUDE M41.GEN
SOLVE M41 USING MCP;

M41.ITERLIM = 2000;

TM2 = 0.05;

$INCLUDE M41.GEN
SOLVE M41 USING MCP;

TM2 = 0.10;

$INCLUDE M41.GEN
SOLVE M41 USING MCP;

TM2 = 0.;
PE1 = 1.2;
PM1 = 1.21;
Exercises:

(1) If you have an international trade textbook, review the Stopler-Samuelson theorem. Examine the effects of the tariff and the terms-of-trade improvement and see that the results validate the theorem. From a policy point of view, the redistribution effects of tariffs are very important and help explain their existence.

(2) Set the tariff to zero and specify an export subsidy on the initially inactive link E2, exports of good 2. How high does this subsidy have to be in order to reverse the direction of trade?

(3) Work out the export tax on $X_1$ that should be equivalent to the import tariff on $X_2$ and verify this numerically (hint: the two tax rates are not same, since the base is different, an issue discussed in the previous chapter).
**Model 42**  Small open economy with a benchmark tariff

This model has a 20% tariff in the benchmark data. It is important to keep track of prices and trade balance in this situation. Here is the data matrix.

### Production Sectors

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Trade balance holds, since exports of good 1 generate 50 units of foreign exchange
and those 50 units are spent on imports. Consumers spend 60 on imports of which 10 is the tariff, or 20% of the value of imports (10/50). This tariff revenue is returned to consumers. All row and column sums equal zero in this matrix.

In open economy models, the modeler must chose units or prices. The convention we adopt here is that all domestic prices are equal to 1 initially. Since the export good \( X_1 \) is freely traded, then the international price of \( X_1 \) is also equal to 1. However, the import good has a domestic price equal to \((1 + TM2)\) times the world price, so if the domestic price is equal to 1 then the world price must equal \( 1 / (1 + TM2) = 1 / 1.2 \).

Thus the -50 in the column M2 of the matrix is interpreted as 60 units at a price of 1/1.2. This is the way the program is formulated below.

The rest of the model should be straightforward at this point. The counterfactual experiment is free trade.
Model M42: Small open economy model with a benchmark *tariff.

In this example, units are chosen such that all DOMESTIC prices equal one initially. Implied world prices are then \( P_1/P_2 = 1.2 \)

### Production Sectors

**Consumer**

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</table>
PARAMETERS

PE2    Export price of good 2,
PM1    Import price of good 1,
PE1    Export price of good 1,
PM2    Import price of good 2,
TM2    Import tariff for good 2;

PE1 = 1;
PM2 = 1/(1.2);
PE2 = PM2*0.99;
PM1 = 1.01;
TM2 = 0.20;

$ONTEXT

$MODEL:M42
$SECTORS:
    X1      ! Production index for good 1
    X2      ! Production index good 2
    E1      ! Export level of good 1
    E2      ! Export level of good 2
    M1      ! Import level of good 1
    M2      ! Import level of good 2
    W       ! Welfare index

$COMMODITIES:
    P1      ! Price index for good 1
    P2      ! Price index for good 1
    PFX     ! Read exchange rate index
    PW      ! Welfare price index
    PL      ! Wage index
    PK      ! Capital rental index

$CONSUMERS:
    CONS    ! Income level for representative agent
$PROD:X1 s:1
  O:P1   Q:150
  I:PL   Q:100
  I:PK   Q: 50

$PROD:X2 s:1
  O:P2   Q:40
  I:PL   Q:20
  I:PK   Q:20

$PROD:E1
  O:PFX  Q: (50*PE1)
  I:P1   Q:50

$PROD:M2
  O:P2   Q:60
  I:PFX  Q: (60*PM2)  A:CONS  T:TM2

$PROD:E2
  O:PFX  Q: (60*PE2)
  I:P2   Q:60
$PROD:M1
  O: P1  Q: 50
  I: PFX  Q: (50*PM1)

$PROD:W  s: 1
  O: PW  Q: 200
  I: P1  Q: 100
  I: P2  Q: 100

$DEMAND:CONS
  D: PW  Q: 200
  E: PL  Q: 120
  E: PK  Q: 70

$OFFTEXT
$SYSINCLUDE mpsgeset M42

PW.FX = 1;
E1.L = 1;
M2.L = 1;
E2.L = 0;
M1.L = 0;

M42.ITERLIM = 0;
$INCLUDE M42.GEN
SOLVE M42 USING MCP;

M42.ITERLIM = 2000;

*          Counterfactual experiment is free trade

TM2 = 0;

$INCLUDE M42.GEN
SOLVE M42 USING MCP;
Exercise:

(1) Verify that an export tax on good 1 has the same effect as an import tariff on good 1 (but remember that the value of the export tax will be different because of the base).

(2) Eliminate the import tariff and replace it with a 20% transport cost. This is modeled by changing the technology in the M2 block so that it takes 20% more PFX to import the same amount of X2. Examine the difference this makes to the welfare cost of the transport cost versus the tariff.
Model M43  Small open economy with a benchmark tariff - alternative price normalization

In models with many countries, the modeler may wish to choose world prices as all equaling 1. Model M43 is exactly the same as model M42 except that we choose world prices as 1. This only affects the calibration via the choice of units for $X_2$. Since the world price of good 2 is equal to 1, then the domestic price is $P_2 = 1.2$. But if this is the case, then the value of $X_2$ production (40) and consumption (100) must imply benchmark quantities of

$$40 = 1.2 \times \text{quantity} \quad \text{quantity} = 33.3333$$
$$100 = 1.2 \times \text{quantity} \quad \text{quantity} = 83.3333$$

You will see these numbers in the quantity fields of the program. W for example, is calibrated as:

```
$PROD:W  s:1
 O:PW    Q:200
 I:P1    Q:100
 I:P2    Q:83.33333  P:1.2
```
Other features of the model are identical to M42 as noted, so here is the program.

$TITLE Model M43: Small open economy model with a benchmark tariff.
* alternative price normalization from M42

$ONTEXT

This model is equivalent to M42 except that units are chosen such that all WORLD prices equal one initially. The benchmark domestic price ratio is then $P_2 = 1.2$.

Note that this changes the units of measurement in good 2. There are now 83.3333 units of good 2 consumed instead of 100, but this is simply a change in units of measure and has no welfare consequences.
# Production Sectors

## Consumer

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<th>Markets</th>
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$OFFTEXT$

## PARAMETERS

- **PE2**: Export price of good 2,
- **PM1**: Import price of good 1,
- **PE1**: Export price of good 1,
- **PM2**: Import price of good 2,
TM2 Import tariff for good 2;

PE1 = 1;
PM2 = 1;
PE2 = 0.99;
PM1 = 1.01;
TM2 = 0.20;

$ONTEXT

$MODEL:M43

$SECTORS:
X1 ! Production index for good 1
X2 ! Production index good 2
E1 ! Export level of good 1
E2 ! Export level of good 2
M1 ! Import level of good 1
M2 ! Import level of good 2
W ! Welfare index
$COMMODITIES:
    P1    ! Price index for good 1
    P2    ! Price index for good 1
    PFX   ! Read exchange rate index
    PW    ! Welfare price index
    PL    ! Wage index
    PK    ! Capital rental index

$CONSUMERS:
    CONS   ! Income level for representative agent

*      Cobb-Douglas production in both sectors:

$PROD:X1 s:1
    O: P1  Q: 150
    I: PL  Q: 100
    I: PK  Q:  50

$PROD:X2 s:1
    O: P2  Q: 33.33333  P: 1.2
    I: PL  Q:  20
    I: PK  Q:  20
$\text{PROD:E1}$

O: PFX  Q: (50*PE1)
I: P1  Q: 50

$\text{PROD:M2}$

O: P2  Q: 50
I: PFX  Q: (50*PM2)  A: CONS  T: TM2

$\text{PROD:E2}$

O: PFX  Q: (50*PE2)
I: P2  Q: (50)

$\text{PROD:M1}$

O: P1  Q: 50
I: PFX  Q: (50*PM1)

$\text{PROD:W}$  s: 1

O: PW  Q: 200
I: P1  Q: 100
I: P2  Q: 83.33333  P: 1.2
$DEMAND:CONS
    D:PW  Q:200
    E:PL  Q:120
    E:PK  Q: 70

$OFFTEXT
$SYSINCLUDE mpsgeset M43

PW.FX = 1;

* Benchmark replication

E1.L = 1;
M2.L = 1;
E2.L = 0;
M1.L = 0;
P2.L = 1.2;

M43.ITERLIM = 0;
$INCLUDE M43.GEN
SOLVE M43 USING MCP;
M43.ITERLIM = 2000;

* Counterfactual experiment is free trade

TM2 = 0;

$INCLUDE M43.GEN
SOLVE M43 USING MCP;
Exercise:

Choices of units with which to interpret the data is a dull, but important step. Many calibration problems (failure to replicate the data as an initial equilibrium) can be traced to mistakes over price and quantity units when there are taxes or quotas in the initial data. So spend some time comparing this file to M42.

Run the models and verify that all results on activity levels and welfare are the same.
Model 44    Small open economy with a benchmark quota

This model assumes the same benchmark data as the previous two models, but it assumes that there is a quota limiting imports. A quota is a quantitative restriction on imports, generally a maximum restriction although there could conceivably be a minimum quota. The former generates a gap between the foreign or world supply price and the domestic demand price for the “rationed” good. This difference, often referred to as a quota rent (it is a form of Ricardian rent), must go to some agent.

In this model, we assume that the government implements the quota via the creation of licenses. These licenses may be auctioned off, sometimes referred to as an auction quota. Or they may simply be given out to some agents inside the economy. Since we have only a single representative household in this model, the model cannot really distinguish between these two. But in a model with several households it can produce an important difference. Indeed, quota licenses are sometimes a source of corruption in real economies, generating rents for favored individuals (the license owners) without them having to produce anything at all.

Using a trick that may now seem familiar from earlier models, we use an auxiliary variable, $Q$, for an endogenous tax rate and a constraint equation to set the value of this “tax”. The revenue from this tax is assigned to the representative consumer in the block
for the import demand for M2.

$\text{PROD:M2}$

\begin{align*}
O & : P2 & Q & : 60 \\
I & : \text{PFX} & Q & : (60*\text{PM2}) & A : \text{CONS} & N : Q
\end{align*}

The constraint equation says to set the value of the tax Q such that the activity level for imports (calibrated to be 1 initially) is equal to 1.

$\text{CONSTRAINT:Q}$

\begin{align*}
1 & =G= \text{M2};
\end{align*}

Other aspects of the model should be familiar. We return here to the convention of model 42 that units are chosen such that all domestic prices equal 1. The counterfactual experiment is to set the quota equal to zero. This is done with the statement $\text{Q.FX} = 0$; since Q is a variable, not a parameter. An alternative way to do this is to declare a parameter and use it in place of the ‘1’ on the left-hand side of the constraint equation. Then the quota can be set at any level.

The value of Q can be thought of as the tariff equivalent of the quota or the “shadow tariff”. In one exercise following the model, you are asked to increase the size of the
economy and see what happens to this shadow tariff. To free up a variable when it has been fixed, use:

\[ Q.L = 0; \]
\[ Q.UP = +INF. \]
In this example, units are chosen such that all DOMESTIC prices equal one initially. Implied world prices are then P1/P2 = 1.2

### Production Sectors

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<tr>
<td>PFX</td>
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<td>-50</td>
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</tr>
</tbody>
</table>
PARAMETERS

PE2  Export price of good 2,
PM1  Import price of good 1,
PE1  Export price of good 1,
PM2  Import price of good 2,
TM2  Import tariff for good 2;

PE1 = 1;
PM2 = 1/(1.2);
PE2 = PM2*0.99;
PM1 = 1.01;
TM2 = 0.20;

$ONTEXT

$MODEL:M42
$SECTORS:
  X1 ! Production index for good 1
  X2 ! Production index good 2
  E1 ! Export level of good 1
  E2 ! Export level of good 2
  M1 ! Import level of good 1
  M2 ! Import level of good 2
  W ! Welfare index

$COMMODITIES:
  P1 ! Price index for good 1
  P2 ! Price index for good 1
  PFX ! Read exchange rate index
  PW ! Welfare price index
  PL ! Wage index
  PK ! Capital rental index

$CONSUMERS:
  CONS ! Income level for representative agent

$AUXILIARY:
  Q
$\text{PROD: X1 s: 1}$

\begin{align*}
O: & \text{P1} \quad Q: 150 \\
I: & \text{PL} \quad Q: 100 \\
I: & \text{PK} \quad Q: 50
\end{align*}

$\text{PROD: X2 s: 1}$

\begin{align*}
O: & \text{P2} \quad Q: 40 \\
I: & \text{PL} \quad Q: 20 \\
I: & \text{PK} \quad Q: 20
\end{align*}

$\text{PROD: E1}$

\begin{align*}
O: & \text{PFX} \quad Q: (50 \times \text{PE1}) \\
I: & \text{P1} \quad Q: 50
\end{align*}

$\text{PROD: M2}$

\begin{align*}
O: & \text{P2} \quad Q: 60 \\
I: & \text{PFX} \quad Q: (60 \times \text{PM2}) \quad \text{A: CONS} \quad \text{N: Q}
\end{align*}

$\text{PROD: E2}$

\begin{align*}
O: & \text{PFX} \quad Q: (60 \times \text{PE2}) \\
I: & \text{P2} \quad Q: 60
\end{align*}
$PROD: M1
  O: P1   Q: 50
  I: PFX  Q: (50*PM1)

$PROD: W  s: 1
  O: PW   Q: 200
  I: P1   Q: 100
  I: P2   Q: 100

$DEMAND: CONS
  D: PW   Q: 200
  E: PL   Q: 120
  E: PK   Q:  70

$CONSTRAINT: Q
  1  =G=  M2;

$OFFTEXT
$SYSINCLUDE mpsgeset M42

PW.FX = 1;
E1.L = 1;
M2.L = 1;
E2.L = 0;
M1.L = 0;
Q.L = 0.20;

M42.ITERLIM = 0;
$INCLUDE M42.GEN
SOLVE M42 USING MCP;
M42.ITERLIM = 2000;

*       Counterfactual experiment is free trade

Q.FX = 0;

$INCLUDE M42.GEN
SOLVE M42 USING MCP;
Exercises:

(1) Free up the quota again (begin with the statements \(Q.L0 = 0; Q.UP = +\text{INF}\); where ‘LO’ stands for lower bound and ‘UP’ stands for upper bound). Double factor endowments. (You can declare a parameter that is multiplied by the factor endowments as in model M1_MPS.) See what happens to the shadow tariff, the value of Q. Can you explain why?

(2) Following this experiment, fix Q at Q = 0.20, its initial value so that it is in fact a tariff. Again double the size of the economy and compare the results of this tariff experiment to the quota experiment.
In some cases, countries impose a so-called voluntary export restraint (VER), which asks a foreign country or foreign firms to limit their exports to a certain quota level. The effect of this is to transfer the quota rents to the foreign country. It is like having a tariff and giving the tariff revenue to the foreign government. In order to model this, we introduce a second consumer, denoted CONSF where F is for foreign and label the domestic consumer as CONSH. CONSF receives the quota rent (shadow tariff revenue) and demands some of the country’s export good, \( X_1 \). Here the numbers we use:
This data is implemented in exactly the manner used in the previous example. We declare an auxiliary variable V, which appears in an N (endogenous tax) field on the import activity, and a constraint equation setting the value of V.

In order to compare this to the auction quota of the previous example, we also declare an auxiliary quota variable Q which is an endogenous tax paid to the domestic consumer, and set by a second constraint equation. In the initial benchmark run of the model, we set Q to zero, Q.FX = 0; and replicate the benchmark. For our first
counterfactual, we replace the VER with the quota using the syntax noted in the previous model:

\[
\begin{align*}
Q\.LO &= 0; \\
Q\.UP &= +\text{INF}; \\
V\.FX &= 0;
\end{align*}
\]

Run the model and examine the results. The equilibria are quite different and the shadow values (Q and V) are not the same. The switch to the auction quota creates an income effect which leads the domestic consumer to demand more goods and more imports at initial prices. But more imports cannot occur under the quota, so in effect the quota becomes more restrictive and the shadow tariff increases (Q is larger than V). Note that the welfare effect of switching from the VER to the quota is quite large.
In this example, units are chosen such that all DOMESTIC prices equal one initially.

<table>
<thead>
<tr>
<th>Markets</th>
<th>X1</th>
<th>X2</th>
<th>E1</th>
<th>M2</th>
<th>W</th>
<th>CONSH</th>
<th>CONSF</th>
</tr>
</thead>
<tbody>
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<td>150</td>
<td>-50</td>
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<tr>
<td>P2</td>
<td></td>
<td>40</td>
<td></td>
<td>60</td>
<td>-100</td>
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</tr>
<tr>
<td>PL</td>
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<td>-20</td>
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<td>120</td>
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<tr>
<td>PW</td>
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<td>190</td>
</tr>
<tr>
<td>PFX</td>
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<td></td>
<td>50</td>
<td>-50</td>
<td></td>
<td></td>
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<tr>
<td>Q</td>
<td></td>
<td></td>
<td></td>
<td>-10</td>
<td></td>
<td></td>
<td>10</td>
</tr>
</tbody>
</table>
PARAMETERS

PE2     Export price of good 2,
PM1     Import price of good 1,
PE1     Export price of good 1,
PM2     Import price of good 2,
TM2     Import tariff for good 2;

PE1 = 1;
PM2 = 1/(1.2);
PE2 = PM2*0.99;
PM1 = 1.01;
TM2 = 0.20;

$ONTEXT

$MODEL:M42
$SECTORS:
    X1    ! Production index for good 1
    X2    ! Production index good 2
    E1    ! Export level of good 1
    E2    ! Export level of good 2
    M1    ! Import level of good 1
    M2    ! Import level of good 2
    W     ! Welfare index

$COMMODITIES:
    P1    ! Price index for good 1
    P2    ! Price index for good 1
    PFX   ! Read exchange rate index
    PW    ! Welfare price index
    PL    ! Wage index
    PK    ! Capital rental index

$CONSUMERS:
    CONSH ! Income level for domestic consumer
    CONSF ! Income level for foreign consumer (quota holder)
$AUXILIARY:
  V      ! Endogenous tax, shadow tax for VER
  Q      ! Endogenous tax, shadow tax for quota

$PROD:X1 s:1
  O:P1   Q:150
  I:PL   Q:100
  I:PK   Q: 50

$PROD:X2 s:1
  O:P2   Q:40
  I:PL   Q:20
  I:PK   Q:20

$PROD:E1
  O:PFX  Q:(50*PE1)
  I:P1   Q:50

$PROD:M2
  O:P2   Q:60
  I:PFX  Q:(60*PM2)  A:CONSF  N:V  A:CONSH  N:Q
$PROD:E2
  O:PFX  Q:(60*PE2)
  I:P2   Q:60

$PROD:M1
  O:P1   Q:50
  I:PFX  Q:(50*PM1)

$PROD:W  s:1
  O:PW   Q:190
  I:P1   Q:90
  I:P2   Q:100

$DEMAND:CONSH
  D:PW   Q:190
  E:PL   Q:120
  E:PK   Q:70

$DEMAND:CONSF
  D:P1   Q:10
$CONSTRAINT: V
   1 =G= M2;

$CONSTRAINT: Q
   1 =G= M2;

$OFFTEXT
$SYSINCLUDE mpsgeset M42

PW.FX = 1;

E1.L = 1;
M2.L = 1;
E2.L = 0;
M1.L = 0;
V.L = 0.20;
Q.FX = 0;

M42.ITERLIM = 0;
$INCLUDE M42.GEN
SOLVE M42 USING MCP;
M42.ITERLIM = 2000;
* Counterfactual: replace the VER with an auction quota

Q.LO = 0;
Q.UP = +INF;
V.FX = 0;

$INCLUDE M42.GEN
SOLVE M42 USING MCP;

Exercise:

Replace the initial VER with a tariff of 0.20 on the import good. Assign the tariff revenue to agent CONSF. Show that this replicates the benchmark data. Show that a tariff of 0.20 assigned to CONSH does not replicated the auction quota calculated in the counterfactual. Why not?
Model M46  Small open economy with a benchmark trade imbalance

Real economies are dynamic and therefore real economies can run trade imbalances in any period by buying or selling assets. Any real data that a modeler is likely to encounter will almost surely have a current account surplus or deficit in any one period. Yet static models are useful for policy analysis and dynamic models are costly to construct in terms of time and data.

When a modeler wants to construct a static model starting with data characterized by a trade imbalance, the model has several choices. These are sometime referred to as “closure rules”. Some closure rules allow the trade balance to vary when doing counterfactuals. The problem with this practice is that it makes welfare effects difficult to interpret. If a policy experiment leads to an increase in the deficit, welfare will increase due, in fact, to foreign borrowing. This is misleading, since at some point that borrowing will have to be paid back.

In general, we prefer to hold the deficit fixed when doing counterfactuals in static models, so as to produce more easily interpreted welfare results. Model M46 shows how to do this.

The simplest trick is to just assume that the representative consumer has an initial
endowment of foreign exchange equal to the deficit. We can think of this as the amount of foreign borrowing in the initial benchmark equilibrium. This is then held constant in any counterfactual experiments. An initial surplus can be represented by a negative endowment of PFX in the benchmark, denoting foreign lending.

This is what we do below. Benchmark exports are 40, imports are 60, and the deficit is financed by selling the initial benchmark endowment of 20 units of PFX. The latter is specified by the parameter BOPDEF which we can change in counterfactual experiments.

$DEMAND:CONS$

<table>
<thead>
<tr>
<th>D : PW</th>
<th>Q : 220</th>
</tr>
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<tbody>
<tr>
<td>E : PL</td>
<td>Q : 120</td>
</tr>
<tr>
<td>E : PK</td>
<td>Q : 80</td>
</tr>
<tr>
<td>E : PFX</td>
<td>Q : BOPDEF</td>
</tr>
</tbody>
</table>

For a counterfactual, we eliminate the deficit, setting BOPDEF = 0. Obviously, welfare is going to fall.

This leads us to a short discussion about another modeling technique. Sometimes, modelers have good reason to believe that the benchmark data is not an equilibrium, or
wish to conduct experiments starting from some other benchmark situation. In such situations, the modeler can compute a “revised benchmark” by eliminating the source of disequilibrium to generate a new benchmark data set. In the present case, if the modeler wanted to start with a situation of a zero balance of trade, the model can then benchmark the model as we have done, and then run the model with BOPDEF = 0. The new equilibrium values of activities, prices, and income then become the new data set or “revised benchmark” for policy analysis.
Model M46: Small open economy model with a benchmark trade imbalance.

Production Sectors

<table>
<thead>
<tr>
<th>Markets</th>
<th>X1</th>
<th>X2</th>
<th>E1</th>
<th>M2</th>
<th>W</th>
<th>CONS</th>
</tr>
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<tbody>
<tr>
<td>P1</td>
<td>150</td>
<td>-40</td>
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<td>-110</td>
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<td>220</td>
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<tr>
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<td>-60</td>
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<td>20</td>
</tr>
</tbody>
</table>


PARAMETERS
   PE2     Export price of good 2,
   PM1     Import price of good 1,
   PE1     Export price of good 1,
   PM2     Import price of good 2,
   TM2     Import tariff for good 2,
   BOPDEF  Balance of payments net deficit;

PE1 = 1;
PM2 = 1;
PE2 = 0.99;
PM1 = 1.01;
TM2 = 0;

$ONTEXT

$MODEL:M46
$SECTORS:
  X1  ! Production index for good 1
  X2  ! Production index good 2
  E1  ! Export level of good 1
  E2  ! Export level of good 2
  M1  ! Import level of good 1
  M2  ! Import level of good 2
  W   ! Welfare index

$COMMODITIES:
  P1  ! Price index for good 1
  P2  ! Price index for good 1
  PFX ! Read exchange rate index
  PW  ! Welfare price index
  PL  ! Wage index
  PK  ! Capital rental index

$CONSUMERS:
  CONS  ! Income level for representative agent
$PROD:X1  s:1
O:P1    Q:150
I:PL    Q:100
I:PK    Q: 50

$PROD:X2  s:1
O:P2    Q:50
I:PL    Q:20
I:PK    Q:30

$PROD:E1
O:PFX    Q:(40*PE1)
I:P1    Q:40

$PROD:M2
O:P2    Q:60
I:PFX    Q:(60*PM2)  A:CONS  T:TM2

$PROD:E2
O:PFX    Q:(60*PE2)
I:P2    Q:60
$PROD:M1
    O:P1   Q:40
    I:PFX  Q:(40*PM1)

$PROD:W  s:1
    O:PW   Q:220
    I:P1   Q:110
    I:P2   Q:110

$DEMAND:CONS
    D:PW   Q:220
    E:PL   Q:120
    E:PK   Q: 80
    E:PFX  Q:BOPDEF

$OFFTEXT
$SYSINCLUDE mpsgeset M46

E2.L = 0;
M1.L = 0;
E1.L = 1;
**Exercise:**

Set BOPDEF to a negative value (a surplus) and see what happens. Interpret the results.
Model 47  Armington formulation

Another feature of real data that confronts modelers is that there is generally two-way trade in any “good” in the data; that is, each good is both imported and exported. This is often attributed to the fact that data is actually classified by industries and any industry is actually an aggregation of many goods. Yet two-way trade or “cross hauling” will persist in the data even at an extremely fine level of disaggregation. No research will ever get data that is free of cross hauling so the question is what to do about it.

Model 47 is calibrated to the following data:

<table>
<thead>
<tr>
<th>Production Sectors</th>
<th>Consumer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Markets</td>
<td>X1</td>
</tr>
<tr>
<td>---------------------</td>
<td>----------</td>
</tr>
<tr>
<td>P1</td>
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<td>-100</td>
</tr>
<tr>
<td>PK</td>
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</tr>
<tr>
<td>PW</td>
<td></td>
</tr>
<tr>
<td>PFX</td>
<td></td>
</tr>
</tbody>
</table>
Each “good” (industry) is both imported and exported. One way to handle this is to simply net out the two way trade from the gross flows and replace the data with net trade only. If we do this in the present model, we arrive exactly at model M41. We can then proceed as before.

The other alternative in competitive models is to assume that the domestic and foreign goods are not really identical. This is the so-called Armington assumption: domestic and foreign goods in the same industrial classification are imperfect substitutes. This is really quite simple for us in MPS/GE. We do not have to do anything on the production side or in the demand blocks. Indeed, all that we have to do is to change the preferences to define four distinct goods and a nesting structure for the utility function. In our case we write this as:

\[
\begin{align*}
\text{PROD:} & \quad W \quad s:1 \quad G1:{\text{ESUB}} \quad G2:{\text{ESUB}} \\
O: & \quad PW \quad Q:200 \\
I: & \quad P1 \quad Q:50 \quad G1: \\
I: & \quad PF1 \quad Q:50 \quad G1: \\
I: & \quad P2 \quad Q:25 \quad G2: \\
I: & \quad PF2 \quad Q:75 \quad G2: 
\end{align*}
\]
The modeler is completely free to choose any nesting structure of course, but this is a common thing to do. The domestic (P1) and foreign good (PF1) in industry 1 are in a nest as are the domestic and foreign goods in industry 2 and these nest have the same elasticity of substitution: a domestic and foreign good in the same industry have the same substitution elasticity regardless of industry. The composite industry 1 good and the composite industry 2 good are then combined in an upper level nest. This is generally assumed to have a lower elasticity of substitution: two industries are poor substitutes than a domestic and foreign good in the same industry. Here we specify the upper nest with an elasticity of substitution of 1, while the domestic-foreign goods have an elasticity of substitution of $\text{ESUB} = 4$.

Other assumptions are possible, such as grouping all foreign goods together in a nest and all domestic goods in another. But the procedure just outlined seems reasonable to many modelers.

The rest of the program is rather familiar, except that the four trade activities are now defined for four distinct goods. There is no need now to worry about a domestic good being both imported and exported. It cannot, by definition, be imported. We run the same counterfactuals that we did for model M41, a tariff of 5% and then 10% on (foreign) good 2.
$TITLE Model M47: Small open economy model with an * Armington formulation.

$ONTEXT

Production Sectors

Consumer

<table>
<thead>
<tr>
<th>Markets</th>
<th>X1</th>
<th>X2</th>
<th>E</th>
<th>M</th>
<th>W</th>
<th>CONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>150</td>
<td>-100</td>
<td>50</td>
<td>-100</td>
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<td></td>
</tr>
<tr>
<td>P2</td>
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<td>75</td>
<td>-100</td>
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<td>125</td>
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</tbody>
</table>

$OFFTEXT
PARAMETERS

PE2     Export price of good 2,
PM1     Import price of good 1,
PE1     Export price of good 1,
PM2     Import price of good 2,
TM2     Import tariff for good 2,
ESUB    Armington elasticity of substitution;

PE1 = 1;
PM2 = 1;
PE2 = 1;
PM1 = 1;
TM2 = 0;
ESUB = 4;

$ONTEXT

$MODEL:M47
$SECTORS:
X1      ! Production index for good 1
X2      ! Production index good 2
E1      ! Export index for good 1
E2      ! Export index for good 2
M1      ! Import index for good 1
M2      ! Import index for good 2
W       ! Welfare index

$COMMODITIES:
P1      ! Price index for good 1
P2      ! Price index for good 1
PF1     ! Price index for imported good 1
PF2     ! Price index for imported good 2
PFX     ! Read exchange rate index
PW      ! Welfare price index
PL      ! Wage index
PK      ! Capital rental index

$CONSUMERS:
CONS    ! Income level for representative agent
* Cobb-Douglas production in both sectors:

$\text{PROD:X1 } s:1$

- $O: P1$  $Q:150$
- $I: PL$  $Q:100$
- $I: PK$  $Q:50$

$\text{PROD:X2 } s:1$

- $O: P2$  $Q:50$
- $I: PL$  $Q:20$
- $I: PK$  $Q:30$

* We scale the export price for good 1 and the import price for good 2 to both be unity:

$\text{PROD:E1}$

- $O: \text{PFX}$  $Q:(\text{PE1} \times 100)$
- $I: P1$  $Q:100$
$PROD:E2
    O:PFX Q:(PE2*25)
    I:P2 Q:25

$PROD:M1
    O:PF1 Q:50
    I:PFX Q:(PM1*50)

$PROD:M2
    O:PF2 Q:75
    I:PFX Q:(PM2*75) A:CONS T:TM2

$PROD:W s:1 G1:ESUB G2:ESUB
    O:PW Q:200
    I:P1 Q:50 G1:
    I:PF1 Q:50 G1:
    I:P2 Q:25 G2:
    I:PF2 Q:75 G2:
$DEMAND:CONS
  D:PW    Q:200
  E:PL    Q:120
  E:PK    Q: 80

$OFFTEXT
$SYSINCLUDE mpsgeset M47

PW.FX = 1;

M47.ITERLIM = 0;
$INCLUDE M47.GEN
SOLVE M47 USING MCP;
M47.ITERLIM = 2000;

TM2 = 0.05;

$INCLUDE M47.GEN
SOLVE M47 USING MCP;
Exercise

Vary the Armington elasticity. Try it at 1.0 and 10.0. See what difference these make to the welfare effects of the tariff in the counterfactuals. What is the economic interpretation of the results.
Model 48    Large open economy

For many countries, or for a small number of commodities for one country, the small-country assumption of fixed world prices may not be appropriate. One alternative is of course to model the whole world. But it may be convenient and appropriate for the modeler to stick with the country and question and continue to represent the rest of the world with trade transformation functions. The purpose of this model is to show how to do this, allowing for prices to change with quantities supplied and demanded.

The powerful features of MPS/GE come with a few costs. One is that production blocks (which are actually cost functions in the background) must be specified with constant returns to scale. Yet having one fixed factor can allow output as a function of one or more variable factors to mimic decreasing returns as we showed in model M25. We will show how to incorporate increasing returns to scale in the next chapter.

We will use a specific factor trick to make the foreign exchange received for exports of $X_1$ a strictly concave function of export volume. We will create a ficticious factor called $R$ (price $PR$) which is a second input into the export function. This factor is owned by a foreign consumer called CONSF and this consumer demands foreign exchange (price $PFX$). In the benchmarking procedure, we scale up the output of export activity $E1$ so that when the ficticious factor is paid the value of its marginal product, the same
amount of foreign exchange is left for the domestic economy as in the original small-
economy benchmarking. Here is the data matrix, which is just a revision of M41, adding
the fictitious factor and scaling up the output of E1 to pay for the factor.
### Production Sectors

<table>
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<tr>
<th>Markets</th>
<th>X1</th>
<th>X2</th>
<th>E1</th>
<th>M2</th>
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Consumer CONSF owns the ficticious factor (price PR) but the amount of foreign exchange left over from exports of good 1, equal to 50, leaves 50 units of FX for the domestic consumer. Here is the production block for activity E1.

\[
\text{\$PROD: E1 s: 1} \\
\text{O: PFX Q: 100} \\
\text{I: P1 Q: 50} \\
\text{I: PR Q: 50}
\]
We have specified this as a Cobb-Douglas function, so we are assuming that it is of the form:

\[ FX = X_1^{0.5} R^{0.5} \]

\[ \frac{X_1}{FX} \frac{\partial FX}{\partial X_1} = 0.5 \]

The elasticity of foreign exchange revenues with respect to export quantity is 0.5. Think of FX as revenue: \( FX = p_1(X_1)X_1 \). Then if you do the algebra, you get:

\[ \frac{\partial FX}{\partial X_1} = p_1 + X_1 \frac{dp_1}{dX_1} \]

\[ \frac{X_1}{FX} \frac{dFX}{dX_1} = 1 + \frac{X_1}{p_1} \frac{dp_1}{dX_1} = 1 + \frac{1}{\eta} \]

where \( \eta \) is the elasticity of demand. If the elasticity of foreign exchange with respect to export quantity is 0.5, then we are implicitly calibrating the elasticity of foreign demand for our exports to be -2. Notice that an infinite elasticity of demand for our exports means that the elasticity of foreign exchange with respect to export quantity is 1, the small country assumption.

Note that by choosing the share parameter on the Cobb-Douglas function \( E_1 \), we can
calibrate to any foreign elasticity of export demand that we like. This is done by varying the amount of R relative to X1 in the benchmarking, remembering to scale output so that there is always 50 units left over for the domestic consumer.

Now run the model. For a counterfactual, we have imposed an import tariff of 0.20. Note from the listing file that this increases welfare in the domestic economy. This is the usual “optimal tariff” argument for a large economy.
## Title

Model M48: Large open economy model: large open economy

## Context

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<th>Markets</th>
<th>X1</th>
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PARAMETERS
   TM2    Import tariff for good;

TM2 = 0;

$ONTEXT

$MODEL:M48

$SECTORS:
   X1    ! Production index for good 1
   X2    ! Production index good 2
   E1    ! Export index of good 1
   E2    ! Export index of good 2
   M1    ! Import level of good 1
   M2    ! Import level of good 2
   W     ! Welfare index

$COMMODITIES:
   P1    ! Price index for good 1
   P2    ! Price index for good 1
   PFX   ! Read exchange rate index
PW  ! Welfare price index
PL  ! Wage index
PK  ! Capital rental index
PR  ! Rent

$CONSUMERS:
CONSH  ! Income level for home agent
CONSF  ! Income level for foreign agent

$PROD:X1 s:1
O:P1  Q:150
I:PL  Q:100
I:PK  Q: 50

$PROD:X2 s:1
O:P2  Q:50
I:PL  Q:20
I:PK  Q:30
$PROD:E1  s:1
  O:PFX  Q:100
  I:P1  Q:50
  I:PR  Q:50

$PROD:M2
  O:P2  Q:50
  I:PFX  Q:50  A:CONSH  T:TM2

$PROD:E2
  O:PFX  Q:(50*0.99)
  I:P2  Q:50

$PROD:M1
  O:P1  Q:50
  I:PFX  Q:(100*1.01)

$PROD:W  s:1
  O:PW  Q:200
  I:P1  Q:100
  I:P2  Q:100
$DEMAND: CONSH
   D: PW   Q: 200
   E: PL   Q: 120
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   D: PFX  Q:  50
   E: PR   Q:  50

$OFFTEXT
$SYSINCLUDE mpsgeset M48

E2.L = 0;
M1.L = 0;

M48.ITERLIM = 0;
$INCLUDE M48.GEN
SOLVE M48 USING MCP;
M48.ITERLIM = 2000;

*   Apply a tariff which improves the terms of trade and
*   home welfare:
TM2 = 0.05;

$\text{INCLUDE M48.GEN}$
SOLVE M48 USING MCP;

Exercises:

(1) Compute the relationship between welfare and tariff rate for different benchmark export demand functions, where the value share of PR in E1 takes on values 0.25 and 0.75. Do this by changing the amount of R in the benchmark, remembering to change output accordingly to leave the return to the domestic consumer equal to 50.

(2) Replace the tariff on good 2 imports with a tax on good 1 exports, and show that you can obtain identical equilibrium values (i.e., demonstrate Lerner symmetry).
Model M49 Two-country world Heckscher-Ohlin model

In some cases, the modeler wants to have a “world” model, or at least a full two-country general-equilibrium model. That is what we present here. It is a textbook, 2x2x2 Heckscher-Ohlin model: two counties, two goods, two factors. The X and Y goods, as they are labeled here, are identical across countries. The factors are labor (L) and capital (K), and the countries are called H (home) and F (foreign).

You now have most of the building blocks necessary for some quite complicated and realistic models. In the present case, there is really nothing new, we are just increasing the dimensions of the problems. Here are the data:
The notation here for the production activities is (industry or good)(country of production)(country of sale). Thus YFH, for example, is good Y produced in country F and sold in country H (exported to country H). The notation for markets and prices should be fairly obvious: PXF for example is the price of good X in country F.

There is free trade, and so there is factor-price equalization. All prices can be set to one. Country H is labor abundant and good X is labor intensive. Country F is capital abundant and good Y is capital intensive.
In our counterfactual experiments, we first allow country H to impose a tariff of 25% and then both countries impose symmetric tariffs of 25%.

$TITLE: \text{Model M49: 2X2X2 HECKSCHER-OHLIN MODEL}$
$* \text{This is a full two-country HO model}$

$ONTTEXT$

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  TARF;

TARH = 0;
TARF = 0;

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$MODEL:M49

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  YHH
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  YHF
  XFF
  YFF
  XFH
  YFH
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  PWF
  PXH
  PXF
  PYH
  PYF
  PLH
  PLF
  PKH
  PKF

$CONSUMERS:
  CONSH
  CONSF

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  I:PLH   Q:120
  I:PKH   Q: 30
$PROD:YHH  s:1
  O:PYH   Q:50
  I:PLH   Q:10
  I:PKH   Q:40

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  I:PLF   Q:40
  I:PKF   Q:10

$PROD:YFF  s:1
  O:PYF   Q:150
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  I:PKF   Q:120

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$OFFTEXT
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  YHF.L = 0.; XFH.L = 0.;

$INCLUDE M49.GEN
SOLVE M49 USING MCP;

*  TARIFFS

TARH = .25;
$INCLUDE M49.GEN
SOLVE M49 USING MCP;

TARH = .25;
TARF = .25;

$INCLUDE M49.GEN
SOLVE M49 USING MCP;

Exercise:

(1) Verify the Stolper-Samuelson theorem by comparing factor prices in free trade versus when one or both countries have tariffs.

(2) See if you can write this model in vector format. Use sets for countries, goods, and factors.
Model 410  International capital flows

Another important feature of economies and also an important policy issue is capital flows or trade in capital. This can be captured in a static model by adding a market for a factor which is simply “rented” from or to foreigners so as to ignore the dynamic and durability issues connected with capital. This model incorporates a simple extension of our earlier work, particularly M48. We are going to assume that the country is a price taker in goods markets, but there is a less-than-perfectly-elastic supply of capital to the country. Once again, we create a fictitious factor R (price PR) which is a fixed factor in a function (KM) transforming foreign exchange into capital and that fictitious factor is owned by a foreign consumer. All in all, the model is very similar to M48, except that now we have three things traded in the benchmark: good 1 is exported, good 2 is imported, and capital is imported. Here is the benchmark data, in which 20 units of capital are imported initially.
Here is the production block for KM:

```$prod:KM s:1
  o:PK q:20
  i:PFX q:10
  i:PR q:10```

The counterfactual experiment is to impose a 5% tariff. You will see that this generates an increase in capital imports, because good 2, the protected import sector is
capital intensive and protection thus raises the return to capital (the Stolper-Samuleson theorem). Welfare of course decreases.

In order to make a simple economics point, we run a second experiment in which we impose the tariff but fix capital imports at their benchmark level of $KM = 1$ (20 units of capital imported are the reference quantity for activity level $KM = 1$). Note that welfare decreases less under this constraint. This is an old result from a paper by Brecher and Alejandro, that capital imports generated by a tariff protecting a capital intensive sector have a secondary, negative welfare effect. Added capital imports allow more costly domestic production of good 2 to displace additional cheap imports, thus creating an added welfare loss.
$TITLE  Model M410: Capital imports (less than perfect elastic)

$ONTEXT

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$OFFTEXT
PARAMETER

  TM2     Import tariff for good X2;

TM2 = 0;

$ONTEXT

$MODEL:M410

$SECTORS:

X1      ! Production index for good 1
X2      ! Production index good 2
E1      ! Export index of good 1
E2      ! Export index of good 2
M1      ! Import level of good 1
M2      ! Import level of good 2
KM      ! Capital imports
W       ! Welfare index
$COMMODITIES:
  P1     ! Price index for good 1
  P2     ! Price index for good 1
  PFX    ! Read exchange rate index
  PW     ! Welfare price index
  PL     ! Wage index
  PK     ! Capital rental index
  PR     ! Rent, implies concavity in capital supply

$CONSUMERS:
  CONSH  ! Income level for home agent
  CONSF  ! Income level for foreign agent

$PROD:X1 s:1
  O:P1   Q:170
  I:PL   Q:120
  I:PK   Q: 50
$PROD:X2  s:1
  O: P2  Q: 50
  I: PL  Q: 20
  I: PK  Q: 30

$PROD:E1  s:1
  O: PFX  Q: 70
  I: P1  Q: 70

$PROD:M2
  O: P2  Q: 50
  I: PFX  Q: 50  A: CONSH  T: TM2

$PROD:E2
  O: PFX  Q: (50*0.99)
  I: P2  Q: 50

$PROD:M1
  O: P1  Q: 50
  I: PFX  Q: (100*1.01)
$PROD:KM  s:1
  O:PK  Q:20
  I:PFX  Q:10
  I:PR  Q:10

$PROD:W  s:1
  O:PW  Q:200
  I:P1  Q:100
  I:P2  Q:100

$DEMAND:CONSH
  D:PW  Q:200
  E:PL  Q:140
  E:PK  Q:60

$DEMAND:CONSF
  D:PFX  Q:10
  E:PR  Q:10

$OFFTEXT
$SYSINCLUDE mpsgeset M410
E2.L = 0;
M1.L = 0;

PW.FX = 1;

M410.ITERLIM = 0;
$INCLUDE M410.GEN
SOLVE M410 USING MCP;
M410.ITERLIM = 2000;

TM2 = 0.05;

$INCLUDE M410.GEN
SOLVE M410 USING MCP;

KM.FX = 1;

$INCLUDE M410.GEN
SOLVE M410 USING MCP;
Exercise:

(1) Place a small tax on imports of capital, with free trade in goods. See what happens to welfare.