Answer Key to Problem Set 4


(5-28) \( Pr(\text{treated}) = 1 - Pr(\text{not treated}) \)

\[ = 1 - Pr(N_1 \cap N_2 \cap N_3 \cap N_4 \cap N_5) \]

(where \( N_i = \text{ith antibiotic does not work} \))

\[ = 1 - Pr(N_1) Pr(N_2) Pr(N_3) Pr(N_4) Pr(N_5) \]

\[ = 1 - (.8)^5 = .67232 \]

This is based on the assumption that whether antibiotic 1 works is unrelated to whether antibiotic 2 works, and so on, for an individual patient. This may be unrealistic.

(5-36) a) \( Pr(S_2 \cap S_1) = Pr(S_2 | S_1) Pr(S_1) = \frac{2}{3} \times \frac{2}{3} = \frac{4}{9} \)

b) \( Pr(F_2 \cap S_1) = Pr(F_2 | S_1) Pr(S_1) = \frac{1}{3} \times \frac{2}{3} = \frac{2}{9} \)

c) \( Pr(\text{at least one survives}) = 1 - Pr(\text{all fail}) \)

For an individual business, the probability it fails within 2 years is

\[ Pr(F_1 \cup (S_1 \cap F_2)) = Pr(F_1) + Pr(S_1 \cap F_2) \]

\[ = Pr(F_1) + Pr(F_2 | S_1) Pr(S_1) \]

\[ = \frac{1}{3} + \frac{1}{3} \times \frac{2}{3} = \frac{5}{9} \]

Thus

\[ Pr(\text{at least one survives}) = 1 - \left(\frac{5}{9}\right)^5 = 1 - \frac{3125}{59,049} = \frac{55,924}{59,049} = .947 \]

This assumes that success or failure is independent across firms. This may not be a good assumption given that failures typically rise during a recession and fall during a recovery.
(6-8) \( \Pr(s \geq 1) = 1 - \Pr(s = 0) = 1 - (.9)^{10} = .6513 \)

(6-12) Let \( E \) = event 2 red and 2 blue marbles

\[
E = (E \cap A) \cup (E \cap B)
\]

\[
\Pr(E) = \Pr(E \cap A) + \Pr(E \cap B) \quad \text{(since A and B are exclusive)}
\]

\[
= \Pr(E \mid A) \Pr(A) + \Pr(E \mid B) \Pr(B)
\]

\[
= \left[\frac{\binom{4}{2}.2^2.8^2}{2!} \cdot .5\right] + \left[\frac{\binom{4}{2}.4^2.6^2}{2!} \cdot .5\right]
\]

\[
= (.1536)(.5) + (.3456)(.5)
\]

\[
= .2496
\]

(6-14) a)

<table>
<thead>
<tr>
<th>( s )</th>
<th>( p(s) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>.3164</td>
</tr>
<tr>
<td>1</td>
<td>.4219</td>
</tr>
<tr>
<td>2</td>
<td>.2109</td>
</tr>
<tr>
<td>3</td>
<td>.0469</td>
</tr>
<tr>
<td>4</td>
<td>.0039</td>
</tr>
</tbody>
</table>

\[
\mu = \sum s \ p(s) = 0(.3164) + 1(.4219) + 2(.2109) + 3(.0469) + 4(.0039) = 1
\]

b) \( n \pi = 4(.25) = 1 \)

If \( \pi = .25 \), then I get a success 25% of the time. If \( n = 4 \), then I'd "expect" one success, so it's not surprising that the mean is 1.
(6-16) a)

<table>
<thead>
<tr>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{6}$</td>
</tr>
<tr>
<td>$\frac{1}{6}$</td>
</tr>
<tr>
<td>$\frac{1}{6}$</td>
</tr>
<tr>
<td>$\frac{1}{6}$</td>
</tr>
<tr>
<td>$\frac{1}{6}$</td>
</tr>
<tr>
<td>$\frac{1}{6}$</td>
</tr>
</tbody>
</table>

b) area under curve = $1(\frac{1}{6}) + 1(\frac{4}{6}) + 1(\frac{1}{6}) = 1$

c) $\text{Pr}(0 < X < \frac{3}{4}) = \frac{3}{4} \times \frac{1}{6} = \frac{3}{24}$

d) $\text{Pr}(X > 2.5) = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$

e) $\text{Pr}(0 < X < 1.5) = 1(\frac{1}{6}) + 1(\frac{4}{6}) = \frac{1}{6} + \frac{4}{12} = \frac{6}{12}$

f) $\text{Pr}(0.5 < X < 2.5) = 1(\frac{1}{6}) + 1(\frac{4}{6}) + 1(\frac{1}{6}) = \frac{1}{6} + \frac{4}{12} + \frac{1}{12} = \frac{5}{6}$