1. A power generating cycle operates between a high-temperature thermal reservoir at temperature $T$ and a lower temperature reservoir of 300 K. At steady state, the cycle develops 50 kW of power while rejecting 1,200 kJ/min of heat energy to the cold reservoir. Find the minimum theoretical value for $T$ in K.

Assumptions: 1. The system undergoes a power cycle at steady state.

An energy rate balance gives

$$\dot{W}_{cycle} = \dot{Q}_H - \dot{Q}_C = 50 \text{ kW}$$

$$\dot{Q}_H = \dot{W}_{cycle} + \dot{Q}_C$$

$$= 50 \text{ kW} \frac{1 \text{ kJ/s}}{1 \text{ kW}} + 1200 \text{ kJ/min} \left( \frac{1 \text{ min}}{60 \text{ s}} \right) = 70 \text{ kJ/s}$$
We know that

\[ \eta \leq \eta_{\text{max}} : \text{that is } \eta = \frac{\dot{W}_{\text{cycle}}}{\dot{Q}_H} \leq 1 - \frac{T_C}{T_H} \]

\[ \frac{50 \text{ kJ/s}}{70 \text{ kJ/s}} \leq 1 - \frac{280}{T_H} \]

\[ \frac{280 \text{ K}}{T_H \text{ K}} \leq 1 - \frac{50 \text{ kJ/s}}{70 \text{ kJ/s}} = 0.286 \]

\[ T_H = 979 \text{ K} \]
2. A air conditioner with a CoP of 2.5 keeps a room at a temperature of 20 °C on a day when the outside temperature was 32 °C. The thermal load at steady state consists of both the energy entering through the walls and windows, at a rate of 31,000 kJ/h and from the occupants, electrical systems and lighting at a rate of 5,000 kJ/h. Determine the power required by this cycle and compare it with the minimum theoretical power required by this refrigeration cycle operating under these temperature conditions (both are in kW).

Assumptions:

1. System undergoes a refrigeration cycle.
2. System operates at steady state.
3. The interior room is the cold reservoir and the outside is the hot reservoir.
4. CoP is 2.5

At steady state the refrigeration cycle must remove energy from the room at the same rate as energy enters from all sources:

\[ \dot{Q}_c = (31,000 + 5,000) \text{ kJ/h} = 36,000 \text{ kJ/h} \]

\[
\text{CoP} = \frac{\dot{Q}_c}{\dot{W}_{\text{cycle}}} \quad \text{thus} \quad 2.5 = \frac{36,000 \text{ kJ/h}}{\dot{W}_{\text{cycle}}} \quad \text{then} \quad \dot{W}_{\text{cycle}} = 14,400 \text{ kJ/h}
\]

or \[ \dot{W}_{\text{cycle}} = (14,400 \text{ kJ/h}) \left( \frac{\text{h}}{3600 \text{ s}} \right) \left( \frac{1 \text{ kW}}{1 \text{ kJ/s}} \right) = 4.0 \text{ kW} \]
We know that the actual CoP must be less than or equal to the CoP of a reversible or Carnot refrigerator cycle operating between these two same thermal reservoirs.

\[
\frac{\dot{Q}_c}{\dot{W}_{\text{cycle}}} = \frac{T_c}{T_H - T_C} = \frac{293}{305 - 293} = 24.42
\]

Thus

\[
36,000 \text{ kJ/h/24.42 (note CoP is dimensionless)} \leq \dot{W}_{\text{cycle}}
\]

or \[
1472.20 \text{ kJ/h} \leq \dot{W}_{\text{cycle}}
\]

and \[
\dot{W}_{\text{cycle}} = (1472.20 \text{ kJ/h}) (1h/3600s) (1 kW/1 kJ/s) = 0.41 kW
\]

The ratio of the actual power required to the minimum theoretical value we have

\[
4.0/0.41 = 9.75
\]

so the actual requirement for the refrigeration unit is almost 10 times greater than the theoretical requirement.

3. A steady state steam power plant transfers steam to the boiler at 210 °C and discharges waste heat to its cooling water at 20 °C. The system requires a work rate of 10,000 kW and operates at a thermal efficiency of 38%. Find the rate that heat energy is discharged by the system and compare it with the minimum theoretical rate.
\[ \dot{W}_{\text{cycle}} = 10,000 \text{ kW} \]

\[ T_H = 210 \, ^\circ \text{C} \]

\[ T_C = 20 \, ^\circ \text{C} \]

\[ \eta = 38\% \]

Assumptions:

1. System is a power cycle or heat engine.
2. Operations are at steady state.
3. Steam is the hot reservoir at 210 °C and the cooling water is at 20 °C.

For a heat cycle at steady state

\[ \dot{W}_{\text{cycle}} = \dot{Q}_H - \dot{Q}_C \quad \text{or} \quad \dot{Q}_C = \dot{Q}_H - \dot{W}_{\text{cycle}}. \]

Also

\[ \eta = \frac{\dot{W}_{\text{cycle}}}{\dot{Q}_H} \quad \text{or} \quad \dot{Q}_H = \frac{\dot{W}_{\text{cycle}}}{\eta} \]

Thus

\[ \dot{Q}_C = \dot{W}_{\text{cycle}} \left[ \frac{1}{\eta} - 1 \right] = 10,000 \text{ kW} \left[ \frac{1}{0.38} - 1 \right] = 16,316 \text{ kW} \]

The thermal efficiency must be less than or equal to the thermal efficiency of a reversible or Carnot process operating between these same two temperatures.

\[ \eta \leq \left[ 1 - \frac{T_C}{T_H} \right] \]

\[ \frac{[10,000 \text{ kW}]}{(10,000 \text{ kW} + \dot{Q}_C)} \leq \left[ 1 - \frac{293}{483} \right] \text{ or } 0.3932 \text{ so that} \]

\[ 10,000 \text{ kW}/0.39 \leq 10,000 \text{ kW} + \dot{Q}_C \quad \text{or} \quad \dot{Q}_C = 15,431.21 \text{ kW} \]

Thus the theoretical minimum rate of energy removed is 15,431.21 kW so the ratio of actual heat to the minimum is

\[ 16,316 \text{ kW}/15,431.21 \text{ kW} \text{ which is very close to 1 so this is a very efficient steam plant.} \]