Example 6.6

PROBLEM  ENTROPY PRODUCTION IN A STEAM TURBINE

Steam enters a turbine with a pressure of 30 bar, a temperature of 400°C, and a velocity of 160 m/s. Saturated vapor at 100°C exits with a velocity of 100 m/s. At steady state, the turbine develops work equal to 540 kJ per kg of steam flowing through the turbine. Heat transfer between the turbine and its surroundings occurs at an average outer surface temperature of 350 K. Determine the rate at which entropy is produced within the turbine per kg of steam flowing, in kJ/kg · K. Neglect the change in potential energy between inlet and exit.

SOLUTION

**Known:** Steam expands through a turbine at steady state for which data are provided.

**Find:** Determine the rate of entropy production per kg of steam flowing.

**Schematic and Given Data:**

![Figure 1:](image)

Figure 1:
We start out with Equation (6-14) from the book

$$\Delta S = \sum \frac{Q}{T} + S_{gen}$$

The problem wants the answer in terms of a rate, so we can dot each term:

$$\Delta \dot{S} = \sum \frac{\dot{Q}}{T} + \dot{S}_{gen}$$

Also, the problem wants the answer per unit kg, so we can divide through by $\dot{m}$

$$\frac{\Delta \dot{S}}{\dot{m}} = \Delta s = \frac{1}{\dot{m}} \sum \frac{\dot{Q}}{T} + \frac{\dot{S}_{gen}}{\dot{m}}$$

Rearranging and removing the summation since there is only heat transfer from one thermal body,

$$\frac{\dot{S}_{gen}}{\dot{m}} = \Delta s - \frac{1}{\dot{m}} \frac{\dot{Q}}{T}$$

Since both states 1 and 2 are completely known (see problem statement and associated $T$-$s$ diagram), $\Delta s$ is known. The surrounding body at temperature, $T$, from where heat transfer is occurring ($Q$) is also known. The only unknown is the $\dot{Q}/\dot{m}$ term. This can be found from the energy equation:

$$Q - W = \Delta E = \Delta KE + \Delta PE + \Delta U$$

The problem statement tells us to neglect any changes in potential energy, so that term vanishes. For the work term, there is obviously shaft work being done. However, there is also flow work since this is an open system and fluid is flowing through it. Recall that the definition for enthalpy is

$$h = u + pv$$

or on a value-change basis as

$$\Delta h = \Delta u + \Delta (pv)$$

Now, for the work term we can write

$$W = W_{\text{shaft}} + W_{\text{flow}}$$

Going back to the energy equation we wrote above, we can bring the flow work onto the right hand side, so now our right hand side is in terms of enthalpy

$$Q - W_{\text{shaft}} = \frac{1}{2} \dot{m}(V_2^2 - V_1^2) + m \Delta h$$

Remember: anytime fluid flows, your energy should be in terms of enthalpy, otherwise it is in terms of internal energy. After rearranging and dotting each term

$$\dot{Q} = \dot{W}_{\text{shaft}} + \frac{1}{2} \dot{m}(V_2^2 - V_1^2) + \dot{m} \Delta h$$
Now, dividing through by \( \dot{m} \),

\[
\frac{\dot{Q}}{\dot{m}} = \frac{\dot{W}_{\text{shaft}}}{\dot{m}} + \frac{1}{2}(V_2^2 - V_1^2) + \Delta h
\]

The values for work per unit mass and the velocities are given in the problem, and the enthalpies can be looked up in the steam tables. This result can be plugged back into the original equation. The final answer is

\[
\frac{\dot{S}}{\dot{m}} = 0.4983 \text{ kJ/kg} \cdot \text{K}
\]