Quantum Many Body Theory

Victor Gurarie

Week 13

13 Weak localization

Long time ago it was observed that the straightforward ladder diagrams summed to derive diffusion (called "diffuson diagrams") are not the only ones. There are also diagrams shown on Fig. 1 which also contribute to the particle's probability to move from one point in space to another. This diagrams correspond to the following expression

$$\delta Y(E,\omega,p) = U^2 \int \frac{d^3q}{(2\pi)^3}\frac{d^3q'}{(2\pi)^3} G^R\left(q+\frac{p}{2}\right) G^R\left(q'+\frac{p}{2}\right) G^A\left(q-\frac{p}{2}\right) G^A\left(q-\frac{p}{2}\right) Y(E,\omega,q+q'). \tag{13.1}$$

Here by δY we denote additional correction to Y, defined in (12.32), additional with respect to (12.33) which was already calculated (with the result (12.39). All the advanced Green's functions are functions of $\omega - E/2$ and all the retarded Green's functions are functions of $\omega + E/2$.

To simply life, we will only calculate δY when $p = 0$. Then things are a little simpler

$$\delta Y(E,\omega,p) = U^2 \int \frac{d^3q}{(2\pi)^3}\frac{d^3q'}{(2\pi)^3} G^R(q)\, G^R(q')\, G^A(q)\, G^A(q')\, \Pi(E,\omega,q+q'). \tag{13.2}$$

Now we only know the expression for Y when $q + q'$ is very small, and it is given then by (12.39). At the same time, q and q' are expected to be of the order of $\sqrt{2m\omega}$. To reflect that, we shift variables of integration $q + q' = k$ and integrate over q and k. In integrating over k, we neglect k in the Green's functions because we expect k to be small. Then we find

$$\delta Y(E,\omega,p) = U^2 \int \frac{d^3q}{(2\pi)^3}\frac{d^3k}{(2\pi)^3} G^R(q)\, G^R(q)\, G^A(q)\, G^A(q)\, \Pi(E,\omega,k). \tag{13.3}$$

The integral over q, after passing to the variable $\xi = q^2/(2m) - \omega$, gives

$$\nu \int \frac{d\xi}{\left(\xi + \frac{i}{2\tau}\right)^2 \left(\xi - \frac{i}{2\tau}\right)^2} = 4\pi\nu\tau^3, \tag{13.4}$$

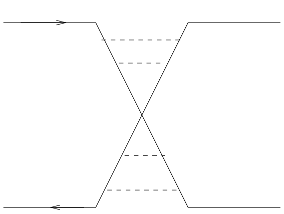

Figure 1: Additional ladder diagrams (Cooperons) which contribute to the calculation of the probability (12.30).

where $\nu = m\sqrt{2m\omega}/(2\pi^2)$ is the density of states (12.36). This gives

$$\delta Y(E, \omega, p = 0) = 4\pi U^2 \tau^3 \nu \int \frac{d^3k}{(2\pi)^3} \frac{2\pi\nu}{Dk^2 - iE} = 2\tau \int \frac{d^3k}{(2\pi)^3} \frac{1}{Dk^2 - iE}. \tag{13.5}$$

Recall that the expression in the integral is only valid at small k. Thus, its divergence at large k is artificial, having to do with the small k approximation. However, its behavior at small k is more interesting. In 3D, this behavior is finite. However, in 2D, if $E = 0$, the integral would have been logarithmically divergent, leading to the answer of the order of $\log(1/E)$. This is very large at small E. So we expect that this expression will result in a large correction to the quantity Y leading to the modification of the diffusive behavior. Same in 1D. What we are observing is the sign of localization, the suppression of diffusion in the spacial dimensions 1 and 2.