Problem Set 3

Phys 7280
Due: Mar 10

1 Polarons at weak coupling

Electrons in a conductance band of a semiconductor form a very dilute gas and as a result, they are not "aware" of the presence of each other. As a result, their noninteracting Green's function is given by

$$G(p,E) = \frac{1}{E - \frac{p^2}{2m} + i0}. \tag{1.1}$$

However, the electrons interact with phonons, causing the distortion of the crystalline lattice and a formation of a new particle, called polaron (an electron with a deformed lattice around it).

The interaction between the electrons and the phonons is given by

$$\hat{H}_{\rm int} = g \int d^3x \, \hat{\psi}^\dagger(x)\hat{\psi}(x)\hat{\rho}(x) \tag{1.2}$$

Here $\hat{\rho}(x)$ is the phonon field operator (it is a boson). The Green's functions of the phonons, defined by $D = -i \langle T \hat{\rho}(x_f, t_f)\hat{\rho}(x_i, t_i)\rangle$, is given by

$$D(k,E) = \frac{c^2k^2}{E^2 - c^2k^2 + i0}\,\theta(k_D - k) \tag{1.3}$$

Here c is the speed of sound, and θ is equal to 1 if its argument is positive and zero if it's negative. k_D is the Debye frequency.

The simplest electronic self-energy diagram is shown on Fig. 1.

1. Write down the expression corresponding to $\Sigma(p,E)$. Show that for $E \approx p^2/(2m)$ and for $p \ll mc$ it takes the form

$$\Sigma(p,E) = \epsilon_0 - \alpha_1 \left(E - \frac{p^2}{2m}\right) - \alpha_2 p^2/(2m). \tag{1.4}$$

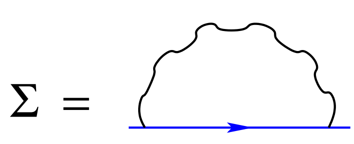

Figure 1: The simplest electronic self energy diagram. The phonons are represented by a wavy line, while the electrons are represented by straight lines.

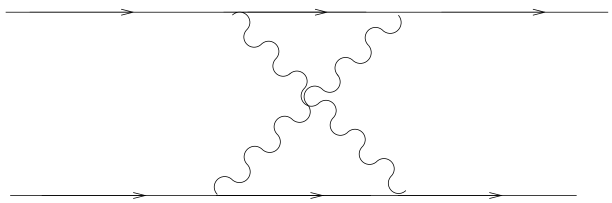

Figure 2: The diagram with crossed interaction lines.

Show that ϵ_0 is the binding energy of this composite particle, while α_2 gives the effective mass of a polaron m^*. Find m^*.
2. Show that if $p > mc$, $\Sigma(p, E)$ is not a real function, but rather a complex function. Calculate Im $\Sigma(p, E)$ and figure out the polaron lifetime.

2 Time dependent interactions

Consider particles which interact via a potential which depends not only on particle separation in space, but also in time (lags in time). For example,

$$U = U(x_1 - x_2)\,\frac{e^{-\frac{|t_1 - t_2|}{2\tau}}}{\tau}. \tag{2.1}$$

Suppose two particles in vacuum scatter in this potential (suppose these particles are distinguishable, for example, two fermions with opposite spin, while the interaction U is only between those opposite spin fermions).

Show that the diagrams which contribute to the scattering of these two particles include diagrams with crossed interaction lines, neglected in class, such as the one shown on Fig. 2. Consider an irreducible diagram (the one which cannot be split in two by cutting *two* particle lines) with N interaction lines, and estimate its dependence on τ as τ is taken to zero. This is easiest to do in the time domain, not in the frequency domain.

3 Heavy particle in a Fermi gas

Suppose a particle of mass M moves in the non-interacting Fermi gas of spin 1/2 fermions of mass $m \ll M$. The fermions do not interact among themselves, but they interact with the heavy particle via a potential $\lambda\delta(x_1 - x_2)$ (instantaneous in time). In the spirit of the Born-Oppenheimer approximation, we can think of light fermions creating an effective potential for the heavy particle.
1. Show that the effects of light fermions on the heavy particle can be thought of, in the second order of perturbation theory, as the effective interaction of heavy particle with itself (which is retarded in time).
2. Calculate that effective potential.