Problem Set 2

Phys 7280
Due: Feb 25

1 Green's functions of a free Fermi gas

The time-ordered Green's function of a free Fermi gas is given by

$$G(p, E) = \frac{1}{E - \frac{p^2}{2m} + \mu + i0 \,\mathrm{sign}\, E}. \tag{1.1}$$

The density of a Fermi system can be calculated using

$$n = -\frac{i}{V} \lim_{t_f \to t_i - 0} \int d^3x \; G(x, x; t_f, t_i), \tag{1.2}$$

where V is the volume and we assume the three dimensional space (thus d^3x in the integral). Use (1.1) and (1.2) to calculate the density of a free Fermi gas in terms of its chemical potential (Fermi energy) μ.

2 The interaction representation

A particle of spin 1/2 and with a magnetic moment μ is subject to a constant vertical magnetic field B_0 and a rotating horizontal magnetic field B_1, so that

$$B = \begin{pmatrix} B_1 \cos \omega t \\ B_1 \sin \omega t \\ B_0 \end{pmatrix}. \tag{2.1}$$

Its Hamiltonian can be written as

$$\hat{H} = \sum_{i=1,2,3} \hat{\sigma}_i B_i, \tag{2.2}$$

where $\hat{\sigma}_i$ are Pauli matrices, and the Hamiltonian acts on two component spinors.
1. Write down the Schrödinger equation in the "interaction representation", considering the time-dependent part of the field a "perturbation".
2. Compute the S-matrix of the problem by solving this Schrödinger equation. An S-matrix is defined as

$$\hat{\psi}(t) = \hat{S}(t)\hat{\psi}(0), \tag{2.3}$$

where $\hat{\psi}(t)$ is the time dependent spinor describing the particle.

3 Scattering in 1D

Consider a particle of mass m moving in a potential $\lambda\delta(x)$ in one dimension. The scattering amplitude $f(k)$ is defined as the appropriate coefficient in the following wave function

$$\psi(x) = e^{ikx} + f(k)\, e^{-ikx},\ x < 0;\ \psi(x) = e^{ikx} + f(k)\, e^{ikx},\ x > 0. \tag{3.1}$$

1. Express $f(k)$ in terms of a T-matrix of this problem. Use the Week 2 notes to adapt the calculations there, done in 3D, to 1D.
2. Express the transmission t and and reflection r coefficients in terms of f. Those are defined by constructing the wave function with the behavior

$$\psi(x) = e^{ikx} + r\, e^{-ikx},\ x < 0;\ \psi(x) = t\, e^{ikx},\ x > 0. \tag{3.2}$$

Hint: Match (3.2) and (3.1) against each other.
3. Use the T-matrix of this problem, computed in the Week 1 notes, and find the transmission and reflection coefficients of the delta-function potential in 1D.