Problem Set 2

Phys 7240
Due: Feb 3

1 Free Particle

The Hamiltonian of a free particle in 1D is given by

$$H = -\frac{1}{2m}\frac{\partial^2}{\partial x^2}. \tag{1.1}$$

1. Compute its Green's function by using the formula

$$G(x, x_0; t) = \sum_n e^{-iE_n t}\psi_n(x)\psi_n^*(x_0) \tag{1.2}$$

2. Check that

$$i\frac{\partial G}{\partial t} = -\frac{\partial^2 G}{\partial x^2},\ \ x > x_0, \tag{1.3}$$

that

$$G(x, x_0; 0) = \delta(x - x_0), \tag{1.4}$$

and that

$$G(x, x_0, t_1 + t_2) = \int_{-\infty}^{\infty} dy\ G(x, y, t_1)G(y, x_0, t_2). \tag{1.5}$$

3. Check that

$$G(x, x_0, N\epsilon) = \left(\frac{m}{2\pi i\epsilon}\right)^{\frac{N}{2}} \int_{-\infty}^{\infty} \prod_{l=1}^{N-1} dx_l\ e^{i\sum_{i=0}^{N-1} m\frac{(x_{i+1}-x_i)^2}{2\epsilon}}. \tag{1.6}$$

2 Harmonic Oscillator

The action for a harmonic oscillator is given by

$$S = \frac{1}{2}\int_0^T dt\ \left[m\dot{x}^2 - \omega_0^2 x^2\right] = \frac{1}{2}\int_0^T dt\ \left\{x\left[-m\frac{d^2}{dt^2} + \omega^2\right]x\right\}. \tag{2.1}$$

In this problem you will calculate the Green's function of the Harmonic oscillator between the points $x(0) = 0$ and $x(T) = 0$, $G(0, 0; T)$, using the approach of operator determinants.

$$G(0, 0; T) \propto \frac{1}{\sqrt{\det\left(-m\frac{d^2}{dt^2} + \omega_0^2\right)}}. \tag{2.2}$$

1. Find the eigenvalues of the operator

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