

Problem Set 5

Phys 7240
Due: Nov 21

Tricritical Ising Model

The tricritical Ising model is defined by its partition function

$$Z = \sum_{\sigma_i = \pm 1, 0} e^{K \sum_{\langle jk \rangle} \sigma_j \sigma_k - \Delta \sum_j \sigma_j^2 + H \sum_j \sigma_j}. \quad (0.1)$$

$\langle jk \rangle$ refers to the nearest neighbors and σ takes values ± 1 and 0 .

1 Tricritical Ising model in one dimension

Compute the partition function above in 1D by transfer matrix technique, at $H = 0$. Introduce $k = e^{-K}$, $\delta = e^{-\Delta}$, and show that at in the limit of zero temperature the free energy has a discontinuity at $k/\delta = 1$. Check that at nonzero temperature the discontinuity disappears, confirming that there are no phase transitions at finite temperature in one dimension. The discontinuity corresponds to the transition between spin up or spin down state and spin zero state.

2 Mean Field Theory

John Cardy, "Scaling and Renormalization in Statistical Physics", problem 2.5.

Do a substitution $\sigma_i = M + (\sigma_i - M)$ in the $\sigma_j \sigma_k$ term and derive mean field theory for the tricritical Ising model, by neglecting terms $(\sigma_j - M)(\sigma_k - M)$ (however, do not neglect $(\sigma_j - M)^2$). Show that the mean field free energy has the form

$$F = KdM^2 - \log \left[1 + 2 \cosh(2KdM) e^{-\Delta} \right], \quad (2.1)$$

where d is the dimensionality of space.

Expand F in Taylor series in M , as in

$$F \approx tM^2 + uM^4 + u_6M^6 \quad (2.2)$$

Show that in the vicinity of the point where u changes sign from positive to negative, u_6 is positive. By minimizing F , calculate the critical exponents in mean field theory.

3 Renormalization Group analysis

John Cardy, "Scaling and Renormalization in Statistical Physics", problem 5.1.

Consider the partition function $Z = \int \mathcal{D}M e^{-F}$, with F given by (2.2).

A tricritical point is where $t = 0$, $u = 0$. Show that the upper critical dimension for the tricritical Ising model is $d_c = 3$ (hint: examine the RG dimension of u_6). Calculate the critical exponents in $\epsilon = 3 - d$ expansion.