

Problem Set 4

Phys 7240
Due: Oct 31

1 Renormalization Group Flow in the 1D Ising model

John Cardy, "Scaling and Renormalization in Statistical Physics", problem 3.1.

Apply renormalization group transformations to the 1D Ising model in the presence of magnetic field. By summing every other spin, define the RG transformation for $b = 2$. Sketch the renormalization group flow diagram in the (x, y) plane where $x = e^{-2K}$, and $y = e^h$.

2 Universality in the 1D Ising model

John Cardy, "Scaling and Renormalization in Statistical Physics", problem 3.4.

By solving the 1D Ising model (without magnetic field) exactly, show that the combination $f_s \xi$ is constant and calculate its numerical value in the limit $T \rightarrow 0$ where ξ diverges. Do the same calculation in the spin 1 Ising model, with the same form for the hamiltonian but with spins s_j taking the values $0, \pm 1$, and verify that the amplitude combination $f_s \xi$ is universal.

3 Properties of the Renormalization Group

John Cardy, "Scaling and Renormalization in Statistical Physics", problem 3.5.

A renormalization group transformation can be written for infinitesimal change of the lattice spacing a , $da = a' - a$, in the vicinity of a fixed point where the couplings $g_i = 0$, in the form

$$a \frac{dg_i}{da} = -\beta_i(g),$$

where $l = \log a$, and β_i are Taylor expandable functions as in

$$\beta_i = -y_i g_i + \sum_{jk} c_{ijk} g_j g_k + \dots$$

Show that, in general, it is possible to define non-linear scaling variables $g'_i = g_i + \sum_{jk} d_{ijk} g_j g_k + \dots$ so that the renormalization group equations simplify to $dg'_i/dl = y_i g'_i$ exactly, with no higher order terms. Under what circumstances does this transformation fail? If there was a nontrivial fixed point at some finite value of the g_i in terms of the old variables, what has happened to it in terms of the non-linear scaling variables?