Sak 19.3.3

# Solutions

$$V(r) = V_0 e^{-r^2/r_0^2}$$

$$f(\theta) = -\frac{2\mu}{\hbar^2}\int_0^\infty \frac{\sin qr}{q} V_0 e^{-r^2/r_0^2}\, r\,dr \qquad q = 2k\sin(\theta/2)$$

$$= -\frac{\mu V_0}{\hbar^2 q}\left[\int_0^\infty \left(\frac{e^{iqr}-e^{-iqr}}{i}\right) e^{-(r/r_0)^2}\, r\,dr\right]$$

$$= -\frac{\mu V_0}{\hbar^2 q}\exp\left(-\frac{r_0^2 q^2}{4}\right)\left[\int_0^\infty u e^{-u^2/r_0^2}\,du + \int_{-\infty}^{\infty}\frac{r_0^2 iq}{4} e^{-(u/r_0)^2}\,du - \int_0^\infty u e^{-(u/r_0)^2}\,du + \int_{-\infty}^{\infty}\frac{r_0^2 iq}{4} e^{-(u/r_0)^2}\,du\right]$$

$$\boxed{\text{where } u \equiv r + \frac{r_0^2 iq}{r}}$$

$$f(\theta) = -\frac{\mu V_0}{i\hbar^2 q} r_0^2 iq \sqrt{\pi}\, r_0\, e^{-r_0^2 q^2/4}$$

$$\boxed{f(\theta) = -\frac{\mu V_0 r_0^3 \pi}{2\hbar^2}\exp\left(-\frac{r_0^2 q^2}{4}\right)}$$

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2 = \mu^2\frac{V_0^2 r_0^6 \pi}{4\hbar^4} e^{-r_0^2 q^2/2}$$

$$\sigma = \iint \frac{\pi r_0^2}{4}\left(\frac{\mu V_0 r_0^2}{\hbar^2}\right)^2 \exp\left(-q^2\frac{r_0^2}{2}\right)\sin\theta\,d\theta\,d\phi$$

$$= \frac{\pi r_0^2}{4}\left(\frac{\mu V_0 r_0^2}{\hbar^2}\right)^2\iint e^{-\frac{q^2 r_0^2}{2}}\sin\theta\,d\theta\,d\phi \qquad q^2 = 2k^2(1-\cos\theta)$$

$$= \frac{\pi r_0^2}{4}\left(\frac{\mu V_0 r_0^2}{\hbar^2}\right)2\pi\int_0^\pi e^{-\frac{r_0^2}{2}\cdot 2k^2(1-\cos\theta)}\sin\theta\,d\theta$$

let $u = (1-\cos\theta)$ $\quad du = \sin\theta\,d\theta$

$$= \frac{\pi^2 r_0^2}{2} \left( \frac{\mu V_0 r_0^2}{\hbar^2} \right)^2 \int e^{-r_0^2 k^2 u} \, du$$

$$= \frac{\pi^2}{2} \left( \frac{\mu V_0 r_0^2}{k^2} \right)^2 \left( 1 - e^{-r_0^2 k^2 \cdot 2} \right)$$

$$\boxed{\sigma = \frac{\pi^2}{2k^2} \left( \frac{\mu V_0 r_0^2}{\hbar^2} \right)^2 \left( 1 - e^{-r_0^2 k^2 \cdot 2} \right)}$$

(2) $$\mathcal{H} = \int d^3k \sum_{\lambda} \left( \frac{1}{2} P^2_{\lambda\vec{k}} + \frac{\omega^2}{2} Q^2_{\lambda\vec{k}} \right)$$

So for any particular $\lambda, \vec{k}$ mode, take functional form:

$$P_{\lambda\vec{k}} = i\omega_k Q^0_{\lambda\vec{k}} e^{i\omega_k t} \qquad Q_{\lambda\vec{k}} = Q^0_{\lambda\vec{k}} e^{i\omega_k t}$$

$$\frac{\partial \mathcal{H}}{\partial P_{\lambda\vec{k}}} = P_{\lambda\vec{k}} \quad , \quad \frac{\partial \mathcal{H}}{\partial Q_{\lambda\vec{k}}} = \omega_k^2 Q_{\lambda\vec{k}}$$

$$= i\omega_k Q^0_{\lambda\vec{k}} e^{i\omega_k t} \qquad\qquad = \omega_k^2 Q^0_{\lambda\vec{k}} e^{i\omega_k t}$$

$$= \frac{dQ_{\lambda\vec{k}}}{dt} \qquad\qquad = -\frac{dP_{\lambda\vec{k}}}{dt}$$

--- which are Hamilton's equations.