Physics 4410 - Quantum Mechanics II – Fall 2004
Problem Set 11

Due November 17, 2004 at 11:00 AM in Duane G2B21

Reading assignment: Shankar Section 18.5; start looking at Chapter 19.

1. Shankar 18.2.2
2. Consider a particle in one dimension moving under the influence of some time-independent potential, $V_0(x)$. Assume that you know the energy levels and corresponding eigenfunctions for this problem. We now subject the particle to a traveling pulse represented by a space- and time-dependent potential,

$$V'(t) = a\delta(x - ct).$$

   a) *(10 pts)* Suppose as $t \to -\infty$ the particle is known to be in the ground state whose wavefunction is $\langle x|i\rangle = \psi_i(x)$. Find the probability for finding the system in some excited state, with wavefunction $\langle x|f\rangle = \psi_f(x)$ as $t \to +\infty$.

   b) *(10 pts)* Reinterpret your result in part (a) as follows. Regard the δ-function pulse as a superposition of harmonic perturbations, by recalling that the δ-function can be represented as a superposition of exponentials:

$$\delta(x - ct) = \frac{1}{2\pi c}\int_{-\infty}^{\infty} d\omega e^{i\omega(x/c-t)}.$$

   Show that if you treat each frequency component of the delta function separately, using for each the result that over long times there is a transition if and only if $\omega = \omega_{fi}$, then you get the same result as in part (a). In other words, we can apply the results for a harmonic time dependence to non-harmonic potentials using Fourier transformations.

   c) *(20 pts)* Apply the result of part (a) to the one dimensional (infinite) square well:

$$V(x) = 0 \qquad 0 < x < L$$
$$V(x) = \infty \qquad x < 0, x > L$$

Find the probability for a transition from the ground state to the first excited state. Explain this in terms of the time it takes light to cross the potential well and the natural timescale of the quantum system.