Liboff 13.29) Pertubation field: $\hat{H}' = 2\hat{\mathbb{H}}'(\vec{r})\cos\omega t$

Let it act for a long time, and let $\omega \simeq \omega_{kl}$.

Since we are dealing with absorbtion,

$$P_{lk} = \frac{4|\mathbb{H}'_{kl}|^2}{\hbar^2(\omega_{kl}-\omega)^2} \sin^2\left[\tfrac{1}{2}(\omega_{kl}-\omega)t\right]$$

Since $\hat{H}'$ acts for a long time, taking the limit $t \to \infty$ we have

$$P_{lk} \to \frac{2\pi t}{\hbar^2}|\mathbb{H}'_{kl}|^2 \delta(\omega_{kl}-\omega)$$

Thus the δ-function indicates that the only state to be excited is the kth state.

The Fourier decomposition of H' in time:

$$\int_{-\infty}^{\infty} \hat{H}' e^{i\omega t}\,dt = 2\hat{\mathbb{H}}'(\vec{r})\int \cos\omega_0 t\, e^{i\omega t}\,dt$$

$$= \hat{\mathbb{H}}'(\vec{r})\int (e^{i\omega_0 t} + e^{-i\omega_0 t})e^{i\omega t}\,dt$$

$$= \hat{\mathbb{H}}'(\vec{r})\int (e^{i(\omega_0+\omega)t} + e^{i(\omega-\omega_0)t})\,dt$$

Recall $\int_{-\infty}^{\infty} e^{i(a+b)t} = 2\pi\delta(a+b)$

$$\Rightarrow \quad = 2\pi\hat{\mathbb{H}}'(\vec{r})\left(\delta(\omega+\omega_0) + \delta(\omega-\omega_0)\right),$$

Picking out $\omega_0 \to -\omega$ or $\omega_0 \to \omega$.

Liboff

13.33 1-D Harmonic Oscillator. Charge to mass e/m, spring constant K in its ground state.

We have $\mathcal{E}(t) = 2\mathcal{E}\cos\omega_0 t$. Thus $V(t) = -2\mathcal{E}\hat{x}\cos\omega_0 t$.
We write $\hat{H}' = -2e\mathcal{E}\hat{x}\cos(\omega_0 t)$

$$= \hat{H}'(x)\, 2\cos(\omega_0 t) \Rightarrow \hat{H}'(x) = -e\mathcal{E}\hat{x}$$

Since $(\omega_{n0} - \omega_0)t \ll 1$ use

$$P_{0n} = \frac{t^2 |\hat{H}'_{n0}|^2}{\hbar^2}.$$ Now, $$H_{n0} = -e\mathcal{E}\langle n|x|0\rangle$$

$$= -e\mathcal{E}\sqrt{\frac{\hbar}{2m\omega_0}}\langle n|\cancel{a}+a^\dagger|0\rangle$$

$$= -e\mathcal{E}\sqrt{\frac{\hbar}{2m\omega_0}}\underbrace{\langle n|1\rangle}_{\delta_{n,1}}$$

Thus

$$P_{0n} = \delta_{n,1}\left|\frac{e\mathcal{E}}{\sqrt{2}\,\hbar\beta}\right|^2 t^2 \quad \text{with } \beta^2 = \frac{m\omega_0}{\hbar}$$

3

Shankar 18.2.1

$$= \sqrt{\frac{\hbar}{2m\omega}}\left(\hat{a}+\hat{a}^{+}\right)$$

$$C_0(t) = \frac{-i}{\hbar}\int_{-\infty}^{\infty} dt\, e^{i\omega t}\left(-e\mathcal{E}\right)\langle 1|\hat{X}|0\rangle \frac{1}{1+\left(\frac{t}{\tau}\right)^2}$$

where $\omega_{10} = \frac{E_1^{(0)}-E_0^{(0)}}{\hbar} = \hbar\omega\frac{\frac{3}{2}-\frac{1}{2}}{\hbar} = \omega$

$$C_0(t) = \frac{-i}{\hbar}\sqrt{\frac{\hbar}{2m\omega}}\left(-e\mathcal{E}\right)\int_{-\infty}^{\infty} dt\, e^{i\omega t}\frac{1}{1+\left(\frac{t}{\tau}\right)^2}$$

$$= \frac{+ie\mathcal{E}\tau^2}{\sqrt{2m\omega\hbar}}\int dt\,\frac{e^{i\omega t}}{t^2+\tau^2}$$

Using integral table: $= \frac{ie\mathcal{E}\tau^2}{\sqrt{2m\omega\hbar}}\frac{\pi}{\tau}e^{-\omega\tau}$

$$P = |C|^2 = \frac{e^2\mathcal{E}^2\tau^2}{2m\omega\hbar}e^{-2\omega\tau} \qquad \text{QED}$$