P4410 LECTURE 34

About to start on scattering theory. Begin with a primer on cross-sections.

The cross-section σ for a process is a measure of the probability of that process happening for a given experiment, per member of the ensemble. Easiest to visualize in terms of a "beam" and a "target", though those concepts aren't necessary in the definition.

Begin with a beam of intensity I — say I protons/cm^2/sec, impinging on a target of n nuclei/cm^3. Note that we're assuming beam and target have uniform density over a (presumably) large area.

As the beam crosses a thickness dx of the target, it will encounter $(n)(dx)(A)$ target particles within a given transverse area A. So because we're assuming everything is uniform across a large area, divide it out:
Target particles encountered$/A$ $= n\,dx$.
The number of interactions in this thickness is $-dI$ (i.e. an interaction attenuates the beam). dI is proportional to:

$$I,\ n,\ dx.$$

Call the proportionality constant σ:

$$dI = -I\sigma n\,dx$$

Dimensions $\frac{\text{particles}}{l^2 t}$ $\quad \frac{\text{particles}}{l^2 t}$ $\quad l^{-3}$ $\quad l$

needs to have l^2 units — ie area.

So we can define σ here without reference to hard spheres or impact parameters.

Note that σ contains all the physics of the interaction! I, n, dx are dependent on how we set up the experiment.

$\boxed{\text{Uses of } \sigma:}$ - Thick target: beam attenuation

$$I(x) = I(0) - \int dx\, dI/dx \quad \text{but} \quad \frac{dI}{dx} = -I\sigma n$$

so

$$I(x) = I(0)\, e^{-n\sigma x}$$

If more than one type of interaction can take place, and dI is the rate of total interactions, then we can break up σ by process:

$$dI = (dI)_a + (dI)_b \quad \text{where } (dI)_a \text{ is \# of type-a interactions, etc.}$$

$$= (-I\sigma_a n\, dx) + (-I\sigma_b n\, dx)$$

$$= -In\, dx\,(\sigma_a + \sigma_b)$$

$\sigma_{a,b}$ partial cross-sec
$\sigma = \sigma_a + \sigma_b$ total cross-sec.

This is trivial to generalize to more than 2 processes:

$$\sigma_{tot} = \sum_i \sigma_i$$

Can further describe the dynamics of the interaction by asking where the particles end up. Consider the number of products scattered into a solid angle segment $d\Omega$ at angles θ, ϕ with respect to beam direction:

$$dI \text{ into } (\theta,\phi) = In\, dx\, d\Omega \frac{d\sigma(\theta,\phi)}{d\Omega}$$

$\frac{d\sigma}{d\Omega}$ = "differential cross-section" — obviously $\int_{4\pi} d\Omega \frac{d\sigma}{d\Omega} = \sigma$.

If multiple particles are created in interaction, may also need to describe dynamics in terms of the outgoing energies:

$$dI\,(\text{into energy } E) = I n \, dx \, dE \, \frac{d\sigma}{dE}$$

If there are multiple particles, angles, etc... then the differential cross-section can become complicated if it describes the full dynamics:

$$\frac{d\sigma}{dE_1 \, dE_2 \, d\Omega_1 \, d\Omega_2 \cdots}\left(E_1, E_2, \theta_1, \phi_1, \theta_2, \phi_2, \ldots\right)$$

Common subatomic cross-section is the "barn" $= 10^{-24}\ cm^2$.

Caveat: if the target causes an adiabatic change to the beam properties (i.e. reduces its energy) then a simple cross-section model is only valid in thin-target limit.

What can σ depend on?

- Incoming beam properties (energy, polarization, charge, mass, ...) but not intensity/density
- Target properties (but not density) — i.e. $V(r)$ if it is a potential well.
- i.e. Properties of the interaction Hamiltonian and kinematics of the beam and target.

Scattering theory: free particle interactions in 3 dimensions.

Start with 1D review: Begin with beam of "monoenergetic" particles $\langle p \rangle = \hbar k_0$ incident from $x = -\infty$, on a potential $V(x)$ that vanishes at $x \to \pm\infty$. For each incoming momentum eigenstate in the beam, a fraction T will be transmitted and $R = 1 - T$ reflected. If incoming p distribution is narrow enough, total R, T depend only on k_0, $V(x)$.