P4410 LECTURE 19

More exam info is on the WWW.

Rules: CLOSED BOOK
NO CALCULATORS / OTHER ELECTRONIC STUFF
1 8½"×11" CRIB SHEET — No need for integrals.
Exam covers material through lecture 18

Hamiltonians w/ spin & $\vec{L}$:

Recall we could treat vector (spinor) field as a direct product space using a scalar field & a 3 (2) - dimensional vector (spinor) space:

$$|\vec{\psi}\rangle = |\psi\rangle \otimes |\chi\rangle$$

where $\hat{\vec{L}}$ generates rotations of $|\psi\rangle$
$\hat{\vec{S}}$ " " " $|\chi\rangle$.

$[\hat{L}_i, \hat{S}_i] = 0$, so:

Can use a basis that consists of eigenvectors in $|\psi\rangle$ space of $\hat{L}^2, \hat{L}_z$ direct product with eigenvectors in $|\chi\rangle$ space of $\hat{S}_z$:

$$|\alpha\, l\, m_\ell\, m_s\rangle = |\alpha\, l\, m_\ell\rangle \otimes |m_s\rangle \quad \text{where}$$

$$\hat{L}^2|\alpha\, l\, m_\ell\rangle = \hbar^2 l(l+1)|\alpha\, l\, m_\ell\rangle$$
$$\hat{L}_z|\alpha\, l\, m_\ell\rangle = \hbar m_\ell|\alpha\, l\, m_\ell\rangle$$
$$\hat{S}_z|m_s\rangle = \hbar m_s|m_s\rangle$$
$$\hat{S}^2|m_s\rangle = \hbar^2 s(s+1)|m_s\rangle$$

but we suppress s since all states have same eigenvalue of $\hat{S}^2$.

...and α refers to any other quantum # we need (i.e. energy) to break degeneracy and specify the basis state.

Example: hydrogen atom, no spin dependence:

$$\hat{H} = \hat{p}^2/2m - \frac{e^2}{r}$$

$$|n\,l\,m_l\rangle \xrightarrow{\text{include spin}} |n\,l\,m_l\rangle\underbrace{|m_s\rangle}_{=\pm\frac{1}{2}} \longrightarrow |n\,l\,m_l\,m_s\rangle$$

Still an $\hat{H}$ eigenstate, degenerate in m_s.
(also of course, and "accidentally" in l.)

Now, apply a "weak" magnetic field: weak enough not to disturb the orbits of the electrons beyond first order.

Now $\hat{H} = \underbrace{\frac{\hat{p}^2}{2m} - \frac{e^2}{r}}_{\hat{H}_E} \underbrace{- \hat{\vec{\mu}}_e \cdot \vec{B}}_{\hat{H}_B}$ where $-\hat{\vec{\mu}}_e \cdot \vec{B} = -\hat{\mu}_z B$ for $\vec{B} = B\vec{e}_z$

What is $\vec{\mu}$? Has L and S contributions:

$$\hat{\vec{\mu}} = -\left(g_l \mu_B \hat{\vec{L}} + g_s \mu_B \hat{\vec{S}}\right) \quad \text{where} \quad \mu_B = \frac{|e|}{2m_e c}$$

and $g_l = 1$, $g_s \approx 2$.

Note - ignore effect of proton spin since $\mu_p \sim \frac{|e|}{2m_p c}$ and $m_p/m_e \sim 2000$

$\longrightarrow$ and of course this means the Hilbert space should be bigger too! (i.e. $|n\,l\,m_l m_s\rangle \otimes |m_{s,\text{proton}}\rangle$)

Now take $\hat{H}_B \cong -\mu_B B\left(-\hat{L}_z - 2\hat{S}_z\right)$

$$\langle E\rangle = \langle n\,l\,m_l\,m_s|\hat{H}_E + \hat{H}_B|n\,l\,m_l\,m_s\rangle$$

These are the unperturbed "original" eigenstates - i.e. $\hat{H}_E$ eigenstates.

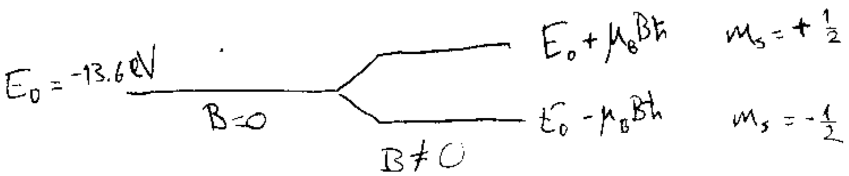
So for each state, $E = E_0 + E'$

Where $E_0 = \frac{-13.6\text{ eV}}{n^2}$ as last semester

and $E' = +\mu_B B\hbar \left[m_\ell + 2m_s\right]$

Consider $n=1$, $\ell=0 \rightarrow m_\ell = 0$

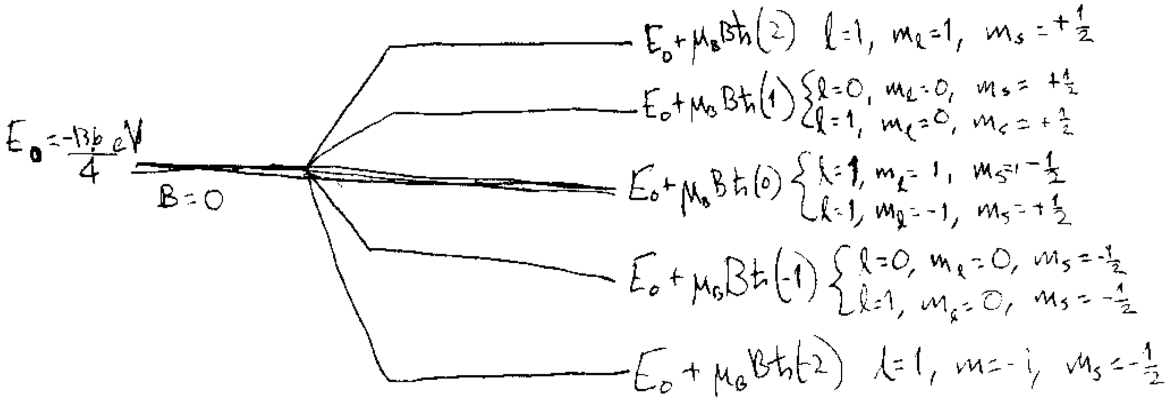
$m_s = \pm\frac{1}{2}$



B-field breaks the 2-fold spin degeneracy.

Consider, $n=2$ 8-fold degeneracy $\left(\begin{array}{l} \ell = 0,\ m_s = \pm\frac{1}{2} \\ \ell = 1,\ m_\ell = +1,\ m_s = \pm\frac{1}{2} \\ \qquad\qquad 0,\ '' \\ \qquad\quad -1,\ '' \end{array}\right)$

How is it broken?



(So there is some remaining degeneracy. But this isn't exact, since $g_s \cong 2.0023$, not 2 — but this is of the same order as proton spin so we are ignoring it.)

This splitting of energy levels is called the ZEEMAN EFFECT.