

Positive Feedback and Oscillators

Purpose

In this experiment we will study how spontaneous oscillations may be caused by positive feedback. You will construct an active LC filter, add positive feedback to make it oscillate, and then remove the negative feedback to make a Schmitt trigger.

Introduction

By far the most common problem with op-amp circuits, and amplifiers in general, is unwanted spontaneous oscillations caused by positive feedback. Just as negative feedback reduces the gain of an amplifier, positive feedback can increase the gain, even to the point where the amplifier may produce an output with no input. Unwanted positive feedback is usually due to stray capacitive or inductive couplings, couplings through power supply lines, or poor feedback loop design. A clear understanding of the causes of spontaneous oscillation is essential for debugging circuits.

On the other hand, positive feedback has its uses. Essentially all signal sources contain oscillators that use positive feedback. Examples include the quartz crystal oscillators used in computers, wrist watches, and electronic keyboards, traditional LC oscillator circuits like the Colpitts oscillator and the Wien bridge, and lasers. Positive feedback is also useful in trigger and logic circuits that must determine when a signal has crossed a threshold, even in the presence of noise.

In this experiment we will try to understand quantitatively how positive feedback can cause oscillations in an active LC filter, and how much feedback is necessary before spontaneous oscillations occur. We will also construct a Schmitt trigger to see how positive feedback can be used to detect thresholds.

Readings

The readings are optional for this experiment. The theoretical background you will need is given in the Theory section below.

1. Horowitz and Hill, Section 5.12 to 5.19. If you are designing a circuit and want to include an oscillator, look here for advice.
2. Bugg discusses the theory of spontaneous oscillations in Chapter 19. You may want to read Section 19.4 on the Nyquist diagram after you read the theory section below.

Theory

Our main goal in this section is to present the general theory for stability of feedback systems, and then to apply the theory to two cases: an op-amp with negative resistive feedback and dominant pole compensation, and the LC oscillator of this experiment. We will show that the dominant pole compensated op-amp is stable for any gain. For the LC oscillator we will find the positive feedback divider ratio B_+ where the system just becomes unstable and the oscillation frequency at threshold. Before going into the stability theory we derive the formulas you will need to design the active LC bandpass filter, and introduce some very useful new mathematics.

THE S-PLANE

Any quantity like a gain $G(\omega)$ or an impedance $Z(\omega)$ that relates an input to an output is called a transfer function. The theory of stability can be presented most easily if we generalize slightly the notion of transfer function that we have been using so far.

When we say that a system has a certain transfer function $T(\omega) = V_{out}/V_{in}$, we mean that if a sine wave with a certain phase and amplitude is applied to the input

$$V_{in}(t) = \text{Re}\{V_{in}e^{i\omega t}\},$$

then the output waveform will be

$$V_{out}(t) = \text{Re}\{T(\omega)V_{in}e^{i\omega t}\},$$

which is a sine wave with a different amplitude and phase. (The input and output do not have to be voltages. For an impedance the output is a voltage but the input is a current.)

It is easy to extend the idea of transfer functions to a more general input waveform:

$$V_{in}(t) = \text{Re}\{V_{in}e^{st}\}.$$

When s is pure imaginary, this is just a sine wave. But in general, s may extend over the whole complex plane, or s-plane, and then we can describe a sine wave multiplied by an exponential function. If we write s in terms of real numbers σ and ω ,

$$s = \sigma + i\omega,$$

then the input waveform becomes

$$V_{in}(t) = \text{Re}\{V_{in}e^{i\omega t}\}e^{\sigma t}.$$

When the real part of s is positive the exponential blows up at late times, but when $\text{Re}\{s\}$ is

negative the wave is damped. With this more general input wave the output is given by

$$V_{out}(t) = \text{Re}\{T(s)V_{in}e^{st}\},$$

The transfer function $T(s)$ on the s -plane is obtained from the frequency domain transfer function $T(\omega)$ by the substitution $i\omega = s$. For example, the frequency domain impedance of an inductor is $Z(\omega) = i\omega L$, and the s -plane impedance is $Z(s) = sL$. You can easily prove this using the fundamental time-domain relation for an inductor: $V(t) = L dI(t)/dt$.

Any transfer function $T(s)$ is a rational function of s , and can be written in the form:

$$T(s) = C \frac{(s - z_1)(s - z_2)(s - z_3) \dots}{(s - p_1)(s - p_2)(s - p_3) \dots}$$

The points on the s -plane z_1, z_2, z_3, \dots where $T(s)$ is zero are called zeros of $T(s)$, and the points p_1, p_2, p_3, \dots where $T(s)$ diverges are called poles of $T(s)$. The prefactor C is a constant. All rational functions of transfer functions, like the numerators and denominators of our gain equations, are also rational functions that can be written in the above pole-zero form.

GAIN EQUATION WITH POSITIVE FEEDBACK

To include in our analysis op-amp circuits like the LC oscillator of Figure 6.3, which have separate paths for positive and negative feedback, we need a more general gain equation. The two divider ratios are defined as before

$$B_+ = \frac{R_2}{R_1 + R_2}, \quad B_- = \frac{R}{R + Z_F}$$

The voltage V_{out} at the output is related to the voltages V_+ and V_- at the op-amp inputs by

$$V_{out} = A(V_+ - V_-)$$

These voltages are themselves determined by the divider networks

$$V_- = V_{in} + B_-(V_{out} - V_{in}),$$

$$V_+ = B_+V_{out}$$

Combining all three relations yields the gain equation

$$G = \frac{V_{out}}{V_{in}} = \frac{-A(1 - B_-)}{1 - A(B_+ - B_-)}$$

In the absence of positive feedback ($B_+ = 0$) this reduces to the gain equation for the inverting amplifier discussed in Experiment #5. (For the non-inverting amplifier, we derived a slightly different gain equation in Experiment #4.)

GENERAL REQUIREMENTS FOR STABILITY

We want to find the conditions for a feedback system to have an output V_{out} even though the input V_{in} is zero. This can happen only where the gain $G(s)$ is infinite, or at a pole of $G(s)$. If all of the poles of $G(s)$ are in the left-half-plane (where $\text{Re}(s) < 0$) then the spontaneous motions are damped and the system is stable. If any pole occurs in the right-half-plane ($\text{Re}(s) > 0$), then there are spontaneous motions that grow with time, and the system is unstable. Poles on the imaginary axis ($\text{Re}(s) = 0$) are an intermediate case where a sinusoidal motion neither grows nor is damped. In this case the system is just at the threshold of oscillation.

All of our gain formulas are of the form $G=N/D$, where N is the numerator and D is the denominator. Poles of G can be due either to poles of N or zeros of D . We will only concern ourselves with cases where the open loop behavior is stable, which is always true for amplifiers but not necessarily for servo systems. Then there are no right-half-plane poles of N , and the stability depends on the presence or absence of right-half-plane zeros of D .

For an amplifier with both positive and negative feedback, our stability criterion is:

If the denominator function $1 - A(B_+ - B_-)$ has any right-half-plane zeros, then the system is unstable.

For an amplifier without positive feedback the criterion is:

If the denominator function $1 + AB$ has any right-half-plane zeros, then the system is unstable.

In the examples below we will determine the presence or absence of right-half-plane zeros directly by solving for the location of the zeros. If this is not possible or convenient, you should be aware that there is another very clever method for determining the presence of right-half-plane zeros called the Nyquist plot (see Bugg, Chapter 19). A different graphical method based on Bode plots is discussed in H&H, Section 4.34. All methods for determining stability are based on the criteria given above.

EXAMPLE: DOMINANT-POLE COMPENSATED OP-AMP

We first consider an op-amp with resistive negative feedback and dominant-pole compensation. This case includes the non-inverting amplifier (Experiment #4, Figure 4.3) and the inverting amplifier (Experiment #5, Figure 5.1). The divider ratio B is frequency independent

$$B = \frac{R}{R + R_F},$$

The frequency-domain open loop gain A of the op-amp

$$A = \frac{A_0}{1 + i \frac{f}{f_0}},$$

corresponds to the s-plane open loop gain A(s):

$$A(s) = \frac{A_0}{s - (-\omega_0)},$$

where $\omega_0 = 2\pi f_0$, and the function has been written in pole-zero form. The denominator function is

$$1 + A(s)B = 1 + \frac{A_0 B}{s - (-\omega_0)} = \frac{s - (-\omega_0(1 + A_0 B))}{s - (-\omega_0)}.$$

For any (positive) value of A_0 or B the denominator function has one zero which is real and negative. Thus there is never a right-half-plane zero, and so a dominant-pole compensated op-amp is stable for any open loop gain (any value A_0) or any closed loop gain (any value of B).

LC ACTIVE BANDPASS FILTER

The circuit for the active LC filter is shown in Figures 6.1 and 6.2. In action, this is very similar to the passive LC filter used in lab 3, except that it has a very low output impedance and depending upon R, may have a very large input impedance. Recall from the theory section of Experiment #5 that the gain of an inverting amplifier is $G = -Z_F/R$ when the open loop gain is large. A little algebra then shows that

$$G(\omega) = -\frac{Z_F}{R} = -\frac{Z_0}{R} \frac{\frac{i}{\omega} + \frac{1}{Q}}{-\frac{i}{\omega} + \frac{1}{Q} + 1},$$

where we have defined the resonant frequency ω_0 , the characteristic impedance Z_0 , and the Q:

$$\omega_0 = \frac{1}{\sqrt{LC}}, \quad Z_0 = \sqrt{\frac{L}{C}}, \quad Q = \frac{Z_0}{r}.$$

With some work you can show that the peak in the magnitude of G occurs at the frequency

$$\omega_{peak} = \omega_0 \sqrt{1 + \frac{2}{Q^2}} = \frac{1}{Q^2},$$

which is very close to ω_0 when Q is large. The gain at the peak is

$$G(\omega_{peak}) = -Q \frac{Z_0}{R}$$

This last formula is only approximate, but corrections to the magnitude are smaller by a factor of $1/Q^2$ and thus not usually important.

LC OSCILLATOR

We consider next the LC oscillator of Figure 6.2 and 6.3, which has a frequency dependent feedback network and both positive and negative feedback. We will have to find the zeros of the denominator function $1 - A(B_+ - B_-)$. If B_+ and B_- were frequency independent, this would be identical to the previous case with the substitution $B = -(B_+ - B_-)$. There would again be one real zero, and it would occur at $\omega = \omega_0 (1 - A_0 (B_+ - B_-))$. The condition for stability would be that this zero be negative, or that $(B_+ - B_-) < 1/A_0$. Thus as soon as the positive feedback exceeds the negative feedback very slightly, the system becomes unstable. (Recall that $1/A_0$ is typically $10^{-4} - 10^{-6}$.) For the problem at hand the solution is more complicated because B_- is frequency dependent.

The zeros can be found by finding the values of s that satisfy the equation $1 - A(B_+ - B_-) = 0$. In our experiment the frequency of spontaneous oscillation is much less than the unity gain frequency of the op-amp. It is therefore an adequate approximation to take the open loop gain A to be infinite, so that the zeros are determined by the simpler equation

$$B_- = B_+$$

This is sometimes called the Barkhausen formula, or when used to find the threshold for oscillation, the Barkhausen criterion. We will see that the real part of this formula can be used to determine the threshold value of B_+ where spontaneous oscillation begins, and the imaginary part determines the frequency of the spontaneous oscillations.

The s -plane impedance of the parallel LC circuit in Figure 6.2 is given by

$$Z_F = Z_0 \frac{\frac{s}{\omega_0} + \frac{1}{\omega_0 Q}}{\frac{s^2}{\omega_0^2} + \frac{s}{\omega_0 Q} + 1}$$

where we have defined ω_0 , Z_0 , and Q as above for the LC active filter. The Barkhausen criterion is then

$$B_- = \frac{R}{R + Z_F} = \frac{\frac{s^2}{\omega_0^2} + \frac{s}{\omega_0 Q} + 1}{\frac{s^2}{\omega_0^2} + \frac{s}{\omega_0 Q} + 1 + \frac{Z_0}{R} \left(\frac{s}{\omega_0} + \frac{1}{Q} \right)} = B_+.$$

Simplifying this result yields a quadratic equation for the zeros:

$$s^2 + \omega_0 \left(\frac{1}{Q} - \frac{B_+}{1 - B_+} \frac{Z_0}{R} \right) s + \omega_0^2 \left(1 - \frac{1}{Q} \frac{B_+}{1 - B_+} \frac{Z_0}{R} \right) = 0.$$

One can easily solve this equation and thereby determine, for any value of the parameters, if there are zeros in the right-half-plane that represent unstable oscillations. We will instead find the value of B_+ where the system just becomes unstable. We expect that for B_+ small enough, all of the zeros should be in the left-half-plane and the system should be stable. Then, as B_+ increases, at least one zero should cross the imaginary axis. The threshold of instability occurs when a zero is on the imaginary axis. We thus look for solutions of the quadratic equation with $s = i\omega$. The imaginary part of the quadratic equation then gives

$$\left. \frac{B_+}{1 - B_+} \right|_{\text{threshold}} = \frac{R}{Z_0 Q} \quad \text{or} \quad \left. \frac{R_2}{R_1} \right|_{\text{threshold}} = \frac{rR}{Z_0^2}.$$

This determines the value of B_+ where the system becomes unstable. The real part yields the oscillation frequency at threshold:

$$\text{threshold} = \omega_0 \sqrt{1 - \frac{1}{Q^2}}.$$

where ω_0 is the natural resonant frequency of the LCR oscillator. For large Q , threshold clearly approaches ω_0 . To find the frequency and exponential growth or decay rate above or below threshold the general quadratic equation for s must be solved.

In real systems, of course, unstable oscillations cannot grow to infinity. Instead they grow until nonlinearities become important and reduce the average positive feedback or increase the average negative feedback. To make a stable oscillator with low distortion, the saturation behavior must be carefully controlled. See the discussion of Wien bridges in H&H Section 5.17 for an example of how this can be done.

Problems

1. Design an active bandpass filter with a resonant frequency of 16 kHz, a Q of 10, and a closed loop gain of one at the peak of the resonance. Choose suitable component values for the parallel LC circuit shown in Figures 6.1 and 6.2, using the inductor that you made in Experiment #3. Use the value of the inductance that you measured earlier. The series resistor shown in Figure 6.2 will have two contributions, one from the losses in your inductor, and one from an actual resistor that you must choose to get the correct Q. If you do not know what the loss of your

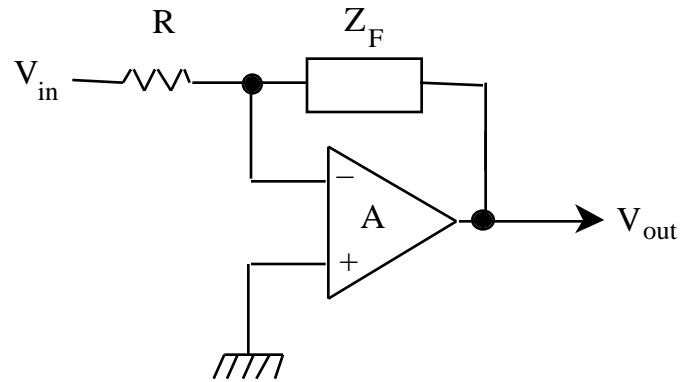


Figure 6.1 Active Bandpass

inductor is, assume it is zero for now.

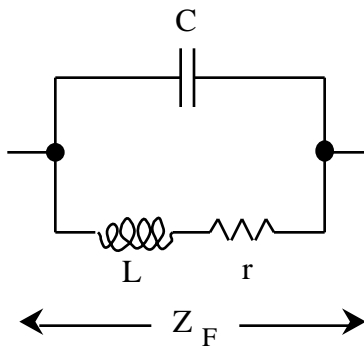


Figure 6.2 Parallel Resonant Circuit

2. To make an LC oscillator you will add positive feedback as shown in Figure 6.3. Predict the value of the divider ratio B_+ where spontaneous oscillations will just begin. B_+ is defined as

$$B_+ = \frac{R_2}{R_1 + R_2}.$$

Also predict the oscillation frequency.

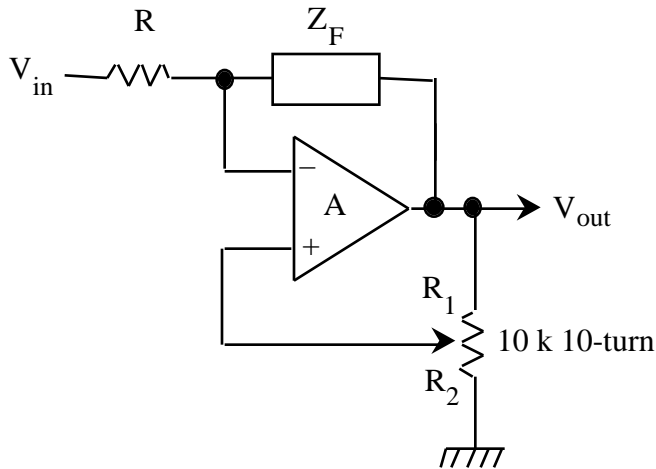


Figure 6.3 Positive Feedback LC Oscillator

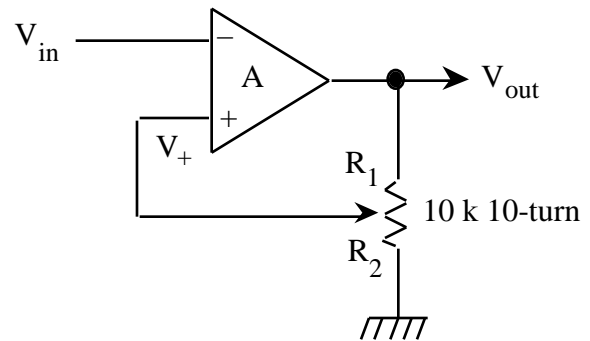


Figure 6.4 Schmitt Trigger

- The circuit shown in Figure 6.4 is called a Schmitt trigger. It has only positive feedback. To figure out what it does, suppose a 1 kHz, 2 V p-p sine wave is connected to V_{in} , and try to draw the waveforms for V_{in} , V_{+} , and V_{out} , all on the same time axis. Suppose that the op-amp saturation levels are ± 13 V, and that the divider is set so $V_{+} = 0.5$ V when $V_{out} = +13$ V.

The Experiment

- Build and test an active LC bandpass filter following the design worked out in problem 1 below. You should try to the closed loop gain on resonance and Q to within 10% of your goals (you may need to adjust r and R). Use your final measurements to refine your values for the circuit parameters (especially L and r, which are hard to measure independently).
- Convert the bandpass filter into an oscillator by adding positive feedback, as described in problem 2. First ground the input ($V_{in} = 0$) and measure the threshold value of the divider ratio at which spontaneous oscillation begins. Measure the oscillation frequency near threshold. Now apply sine waves to the input and observe the output as a function of input for various frequencies and various values of the positive feedback divider ratio B_{+} . Describe your findings.
- Remove the negative feedback from your circuit to create a Schmitt trigger. Compare the observed waveforms with those predicted in problem 3.