Filters And Waveform Shaping

Purpose

The aim of this experiment is to study the frequency filtering properties of passive (R, C, and L) circuits for sine waves, and the transient response of the same circuits to square waves. Filters are important in experiments for enhancing signals of particular interest while suppressing unwanted background.

New techniques in this experiment include ferrite inductors and the oscilloscope probe.

Introduction

A frequent problem in physical experiments is to detect an electronic signal when it is hidden in a background of noise and unwanted signals. The signal of interest may be at a particular frequency, as in an NMR experiment, or it may be an electrical pulse, as from a nuclear particle detector. The background generally contains thermal noise from the transducer and amplifier, 60 Hz power pick up, transients from machinery, radiation from TV stations, and so forth. The purpose of filtering is to enhance the signal of interest by recognizing its characteristic time dependence and to reduce the unwanted background to the lowest possible level.

In everyday life, your radio does this when you tune to a particular station, using a resonant circuit to recognize the characteristic frequency. The signal you want may be less than 10^{-6} of the total radiation power at your antenna, yet you get a high quality signal from the selected station.

A filter freely transmits electrical signals within a certain range of frequencies called the pass-band, and suppresses signals at all other frequencies (the attenuation bands). The boundary between a pass band and an attenuation band is called the cut-off frequency f_c. We usually define f_c to be the frequency at the half-power point or 3dB point, where the power transmitted is half the maximum power transmitted in the pass band. The output voltage amplitude at $f = f_c$ is $1/\sqrt{2} = 70\%$ of the maximum amplitude.

There are three basic types of filter. The high-pass filter transmits signals at frequencies above the cut-off f_c and blocks them at lower frequencies. The low-pass filter transmits signals below f_c and blocks higher frequencies. The band-pass filter, often taking the form of a resonant circuit, transmits a certain band of frequencies and blocks signals outside that band. For band-pass filters the bandwidth is the range of frequencies between the upper (f_+) and lower (f_-) half power points: bandwidth $\equiv \Delta f = f_+ - f_-$.

Many experiments require specific filters designed so that the signal from the phenomenon of interest lies in the pass-band of the filter, while the attenuation bands are chosen to suppress the background and noise.

This experiment introduces you to the filtering properties of some widely used but simple circuits, employing only a resistor and capacitor for high- and low-pass filters and an LCR circuit for band-pass.

Readings

1. D&H Chapters 2 and 3
2. H&H Appendix A on the oscilloscope probe.
3. (optional) D.V. Bugg, Chapter 14. The theory of series and parallel LCR circuits is presented in detail.

Theory

RC FILTERS

The response of RC low-pass and high-pass filters to sine waves is discussed in D&H chapters 2 and 3. The important formula for both cases is

$$\boxed{f_c = \frac{1}{2\pi RC}}$$

where f_c is the 3 dB or half-power point. The response of RC circuits to square waves (steps) is also discussed. The main result is that the exponential rise and decay times t_R and t_D are equal to RC.

LCR RESONANT CIRCUIT

For the parallel LCR band-pass circuit with sine wave input, the resonant frequency and Q are given by

$$\boxed{f_0 = \frac{1}{2\pi\sqrt{LC}} \qquad Q = \omega_0 RC}$$

where $\omega_0 = 2\pi f_0$.

The transient or step response of an LCR circuit, and the sine-wave response as well, are covered in D.V. Bugg, Chapter 14. Bugg derives a differential equation for the current I_1 through the inductor (Bugg Eqn. 14.31):

$$\frac{V}{CLR} = \frac{d^2 I_1}{dt^2} + \frac{1}{CR}\frac{dI_1}{dt} + \omega_0^2 I_1$$

The general solution to this equation is

$$I_1 = e^{-\gamma t}\left(A\cos\omega_1 t + B\sin\omega_1 t\right) + \frac{V}{R}$$

where

$$\boxed{\gamma = \frac{1}{2RC}, \qquad \omega_1 = 2\pi f_1 = \sqrt{\omega_0^2 - \gamma^2}, \qquad \omega_0 = 2\pi f_0.}$$

At the rising voltage step, the boundary conditions are $I_1(0) = 0$ and $dI_1/dt = 0$. This leads to the solutions

$$A = -\frac{V}{R}, \qquad B = -\frac{V}{2\omega_1 R^2 C}.$$

Our measured V_{out} is the voltage across the inductor, which is given by

$$V_{out} = L\frac{dI_1}{dt}.$$

Putting everything together we find the result

$$\boxed{V_{out} = V_{in}\frac{1}{\omega_1 RC}e^{-\gamma t}\sin\omega_1 t}$$

The response is a sine-wave with an exponentially decaying envelope or amplitude. The exponential decay time of the envelope is $1/\gamma$. The natural frequency f_1 can be found by measuring the time t_1 between zero-crossings, and then $f_1 = 1/t_1$.

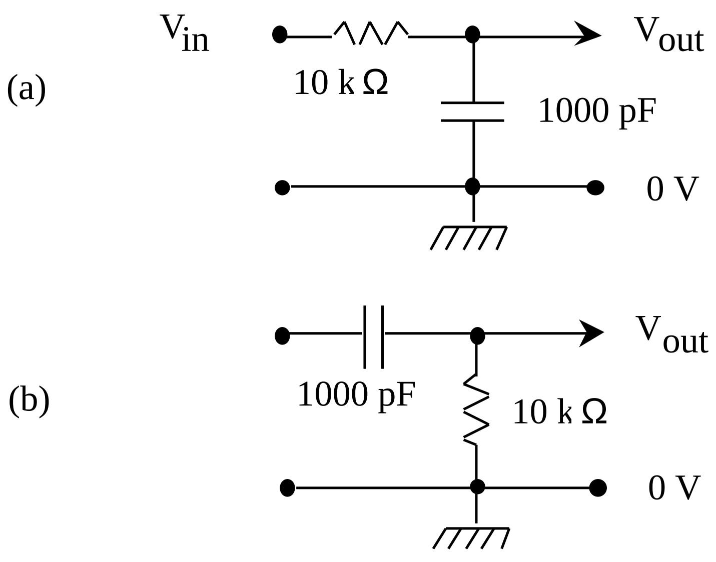


Figure 3.1

Problems

1. Calculate the cut-off frequency f_c for the low-pass filter in Figure 3.1 (a). Draw a Bode diagram (log-log plot of Transfer function T vs. f) of the frequency response, sketching in the correct form near the half power point. Calculate the integrating time constant t_R. Draw the output waveform where the input is a low-frequency square wave.

2. Calculate the cut-off frequency for the high-pass filter in Figure 3.1 (b) and draw its Bode diagram. Calculate the differentiating time constant t_D and draw the output waveform for input square waves.

3. For a resonant circuit the <u>characteristic impedance</u> Z_0 is the magnitude of the impedance of the inductor or the capacitor at the resonant frequency. Calculate the resonant frequency f_0, the characteristic impedance Z_0, and the quality factor Q for the band-pass filter in Figure 3.2. Make a Bode plot showing the expected attenuation versus frequency and mark on it the two half power points f_+ and f.

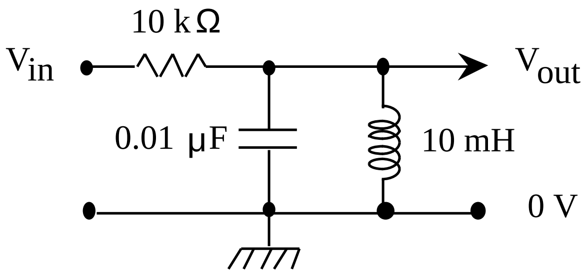


Figure 3.2

· **Outline of the Experiment**

1. Introduction to ferrite inductors.

2 Assemble the following circuits on your board: a resistive divider, a high-pass filter, a low-pass filter, and a resonant band-pass filter. Using the measured component values, construct Bode diagrams (log-log plots of attenuation versus frequency) for each circuit.

3. Using sine waves, measure the attenuation and phase shift as a function of frequency for each circuit.

   Determine the cut-off frequencies f_c for the high- and low-pass RC filters, and compare with predictions. Test your theoretical Bode diagrams by superposing experimental points on them. Explain the surprising high frequency cut-off for the resistive divider. (Consider the capacitance of the cable).

   Repeat the resistive divider using the oscilloscope 10x probe and explain your result.

   Measure the resonant frequency f_0, and the quality factor Q for the band-pass filter. The quality factor is a measure of the sharpness of the resonance. The higher the value of Q, the sharper the resonance.

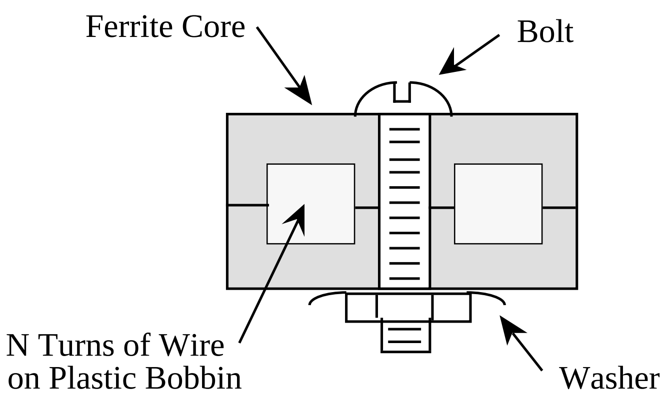
4. With low frequency square waves, measure the exponential rise time t_R for the resistive divider and low-pass filter (integrator), and the decay time t_D for the high-pass (differentiator) circuit. Compare with predictions and with the cut-off frequencies measured with sine waves.

   For the resonant band-pass circuit, measure the natural frequency f_1 and decay rate γ for the transient oscillations excited by the step in the square wave. Unless the Q is very large, the natural frequency of free oscillations f_1 will be less than the driven resonant frequency f_0. Are f_1 and γ consistent with the values of f_0 and Q measured with sine waves?

New Apparatus And Methods

A typical layout for the RLC bandpass filter is shown in Figure 2.3. Design the layout of your circuit to look like the schematic diagram, so far as is possible. This will help you minimize wiring mistakes and makes trouble shooting easier.

INDUCTOR
Make up your own 10 mH inductor from the kit of parts. Keep it for the semester to use in other experiments. The inductor is a ferrite pot-core type that is commonly used at frequencies below 1 MHz. The particular core is # 1811-L00-3B7.



For this core, we can estimate the inductance with the formula:

$$L = N^2 \bullet 3\ \mu H$$

Figure 3.3   Ferrite Inductor

Ferrite is a ferromagnetic oxide of iron of high magnetic permeability formed into a ceramic. It can be used at much higher frequencies than ordinary iron cores because its high electrical resistivity suppresses the eddy currents that cause energy loss in iron metal cores.

You will need to wind about 58 turns of 28 gauge enameled wire on the plastic bobbin to obtain L=10 mH. There could be a 10% variation in L depending on the permeability for the particular specimen of ferrite. You will likely find a slight frequency dependence to your measured value of L. Take note of this dependence when making your measurements.

You will need to solder 22 gauge wire ends onto the 28 gauge magnet wire to make good contacts on the board. Remember to clean the enamel off the ends before soldering.

The experiment

THE CIRCUITS

Make up the following circuits on your board (figure 3.4):

a) Resistive divider.
   R = 10 kΩ
   R = 6.8 kΩ

b) Low-pass filter
   R = 10 kΩ
   C = 1000 pF

c) High-pass filter
   R = 10 kΩ

C = 1000 pF

d) <u>Bandpass filter</u>

R = 10 kΩ

C = .01 µF

L = 10 mH

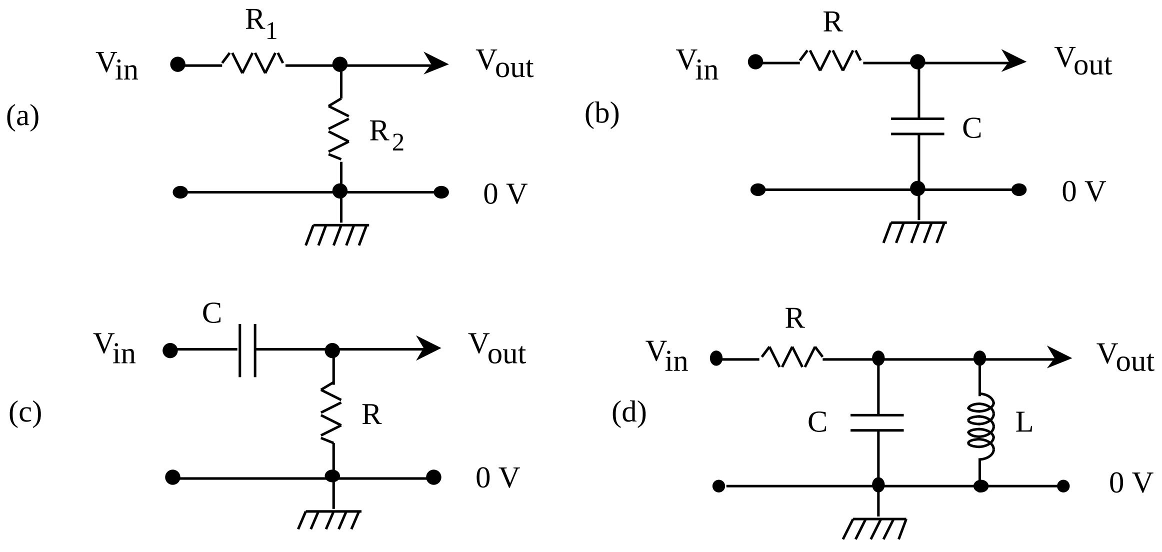


Figure 3.4

If you cannot find components in stock with the specified values, take the nearest in value that you can find, within 30% if possible.

- Measure the value of each resistor and capacitor before you insert them. Why before?
- Calculate the expected attenuation of the divider. Calculate the expected values of the cut-off frequencies for the high- and low-pass filters using the actual component values. Calculate the expected resonant frequency f_0 and quality factor Q for the band-pass filter using the actual component values.
- Draw the Bode diagrams for each circuit.

TEST SET-UP

Connect the circuit board to the function generator, the oscilloscope and the counter/timer as shown in Figure 3.5.

- Set the oscilloscope to display CH 1 and CH 2 using sweep A triggered by CH 4 (not displayed). Use dc coupling.
- Set the counter-timer to measure frequency to 10 MHz.
- Test the system with 1 kHz sine waves at 1 volt p-p. Display CH 1 and CH 2 waveform on the screen.

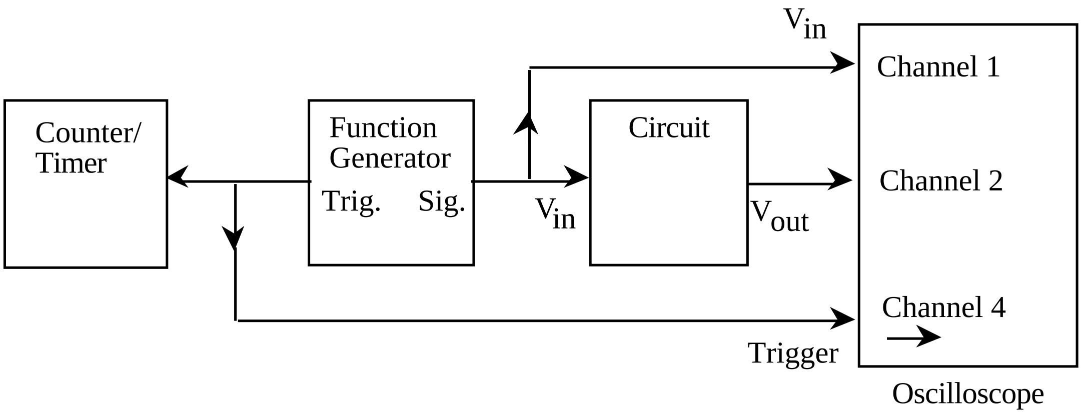


Figure 3.5 Test Set-up

MEASUREMENTS WITH SINE WAVES

First make connections to the voltage divider. Vary the frequency over a wide range with the function generator. Verify that the attenuation is independent of frequency up to a very high frequency. Compare the measured value of attenuation with that predicted from component values.

- What is the phase difference between V_{in} and V_{out}? Is it reasonable?
- If there is a high frequency cut-off, measure its value and explain quantitatively. (Consider the capacitance of the cable and the scope input).
- Repeat using the oscilloscope 10x probe to observe the output of the circuit. Explain your observation.

Use the 10x probe to measure output on all remaining measurements.

Next study the high-pass and low-pass filters. Determine the frequency at the half power point f_c for each filter. Compare with the cut-off frequency computed from the component values.

- Measure the attenuation vs. frequency at decade intervals from $f = 10^{-3} f_c$ to $f = 10^3 f_c$ if possible. Leave the dial at f_c, and use the multiplier switch to change decades. Test the predicted frequency response by marking your data points directly on your Bode diagram.
- Measure the phase shift between input and output sine waves at $f = 0.1 f_c$, f_c, and $10 f_c$. Mark these data on a phase diagram. (See D&H Fig. 3.9b for an example of a phase diagram.)

Finally look at the band-pass filter. Find the resonant pass frequency f_o two ways.

- Adjust the frequency so that:
  a) The output has maximum amplitude (V_{out}/V_{in}=max).
  b) There is zero phase difference between V_{out} and V_{in}.
- Which method is the more precise?
- Determine the quality factor Q by measuring the frequencies at the two half-power points f_+ and f_- above and below the resonance at f_o. The quality factor is defined by

$$Q = \frac{\text{Resonant frequency } f_0}{\text{Bandwidth } \Delta f}$$

where $\Delta f = f_+ - f_-$.

- Map out the shape of the resonance curve by measuring the attenuation at f_o, $f_o \pm \Delta f/4$, $\pm \Delta f/2$, $\pm \Delta f$, $\pm 2\Delta f$, $\pm 5\Delta f$.
- Compare the measured f_o with the theoretical value $1/\left(2\pi\sqrt{LC}\right)$. There could be up to 10% error due to uncertainty in the value of L. Calculate the true value of L from the measured values of f_o and C, and use this value in the future.
- Compare the measured value of Q with that predicted from measurements of component values.
- Test the predicted frequency response by plotting your data on your predicted resonance curve.

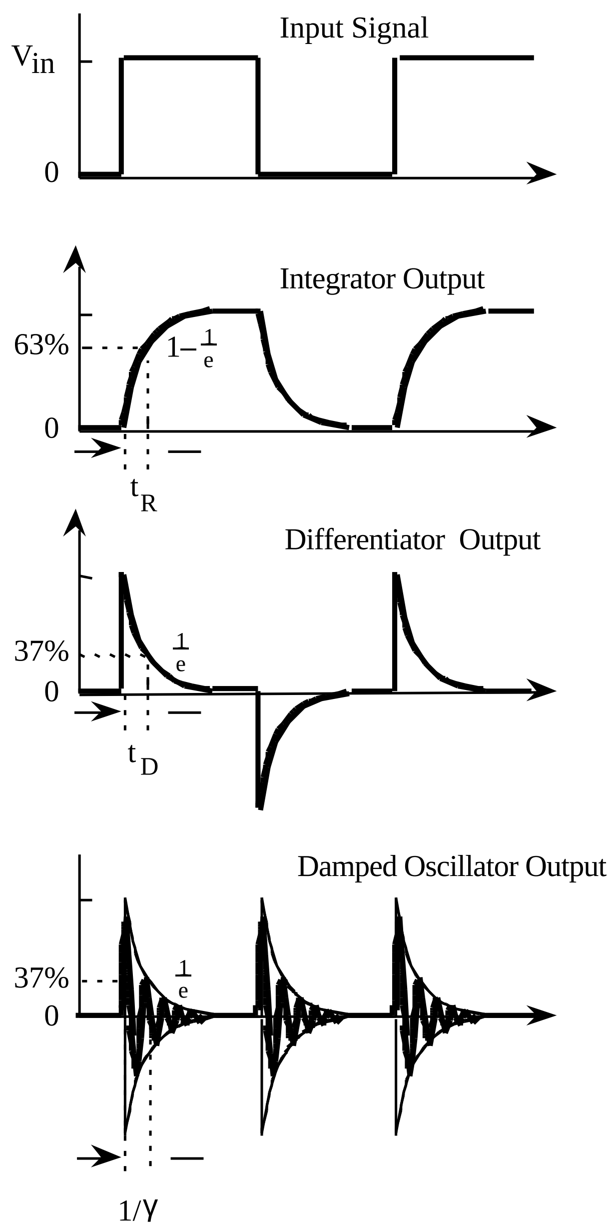


Figure 3.6

MEASUREMENTS WITH SQUARE WAVES

Examine the transient response excited by the voltage step at the beginning of each square wave. Transient responses indicate how each circuit will affect information in the form of pulses such as those produced by particle and photon detectors. When used to process pulses:

- A <u>low-pass filter</u> is called an <u>integrator circuit</u>.
- A <u>high-pass filter</u> is called a <u>differentiator circuit</u>.
- A <u>band-pass filter</u> is called a <u>damped oscillator</u>.

Change the function generator output to square waves 1 V p-p with risetime fixed (a few ns). Display waveforms for V_{in} and V_{out}.

- Reduce the frequency until the interval between steps is long enough to allow transients to decay within 1% of final voltage before the next step.

- Use cursors for accurate measurement of the quantities t_R, t_D, γ, and f_1 (see below).
- Example—to measure t_R for integrator:
  a) Measure V_{in} of input wave.
  b) Set voltage cursor at 63% V_{in}.
  c) Identify intersection of cursor with wave.
  d) Use time cursors to measure interval from start of transient to intersection at 63%.

For the divider, integrator, and differentiator circuits:

- Measure exponential rise-time t_R for the divider and integrator. Measure the decay-time t_D for the differentiator.
- Do the results satisfy the expected relations?

a) t_R = RC for the integrator, and t_D = RC for the differentiator.

b) f_c (low pass) = $(2\pi t_R)^{-1}$ and f_c (high pass) = $(2\pi t_D)^{-1}$ (Where the cut-off frequencies f_c are the values measured from attenuation of the sine waves.)

For the resonant band-pass filter:

- Observe the waveforms and check that the transient has decayed to less than 1% between the steps of the square wave.
- Measure the <u>natural resonance frequency</u> f_1 using the time cursors. Take your measurements using baseline crossings rather than at peaks (why?). If you measure the perod t_1 (the time between <u>every other</u> zero crossing), then $f_1=1/t_1$. Measure about 5 periods for better accuracy. How does f_1 compare with f_0?

Measure the <u>exponential decay rate</u> γ of the transient amplitudes. γ is defined as the reciprocal of the exponential decay time of the envelope function. You may do it roughly by eye using the screen scale. Theoretically, we expect $Q=\omega 0/(2\gamma)$, where $\omega 0=2\pi f0$. How close is this?