Lecture 5

Ohm's Law

It states that for most substances, the current density is proportional to the force per unit charge, \( \mathbf{F} \)

\[
\mathbf{J} = \sigma \mathbf{E}
\]

\( \mathbf{F} = (E \times \mathbf{v} \times \mathbf{B}) \)

\( \text{Ordinary, } |vB| \ll |E| \)

\[
\mathbf{J} = \sigma \mathbf{E}
\]

\( \sigma \) - conductivity

for a perfect conductor \( \sigma \rightarrow \infty \Rightarrow \mathbf{E} = 0 \)

even \( \mathbf{J} \) is not zero

\[
\sigma = \frac{1}{\rho} \text{ resistivity}
\]

Imagine a wire with uniform current

\[
I = JA = \sigma EA = \sigma AV
\]

\[
V = \frac{L}{\sigma A} I
\]

\[
R = \frac{L}{\sigma A} \text{ resistance}
\]

\( \Omega \) - ohms

\( 1 \Omega = 1V/A \)
But wait, if the force is \( eE \), this in principle would accelerate the charges leading to a time increasing current. Why is there a constant on time?

The reason is that electrons in a conductor are always colliding with the atoms. This collision is the leading mechanism that causes the resistivity.

Note that in the absence of an electric field there is no current since the average (thermal) velocity of the electrons is zero: 
\[ \langle V_{th} \rangle = 0 \]

When there is an electric field beside the random thermal motion there is a small drift velocity (\( \sim 10^\text{-9} \text{m/s} \)). Note that the mean magnitude of the thermal velocity is 
\[ V_{th} = \langle V_{th} \rangle \sim 10^\text{6} \text{m/s} \] so 
\[ V_d \ll V_{th} \]

Let's calculate \( V_d \): since 
\[ F = ma \] then 
\[ a = \frac{eE}{m} \]
this acceleration only acts in the short time between collisions. If $\tau$ is the mean time between collisions and $\bar{v}$ to the initial velocity, then the velocity before the next collision is

$$\bar{v} = \overline{v_0 + g \bar{E} \tau}$$

Taking the average

$$\langle \bar{v} \rangle = \bar{v}_d = \frac{g \bar{E} \tau}{2m}$$

Now $\tau = \frac{2}{\bar{v}_m}$ with $\bar{v}$ the mean free path

$$\bar{v}_d = \frac{g \bar{E}}{2m} \frac{2}{\bar{v}_m}$$

If there are $n$ atoms per unit volume and 4 free electrons per atom

$$\bar{\sigma} = n f g \bar{v}_d = \left( \frac{4 g^2 A}{2m \bar{v}_m} \right) \bar{E}'$$

Note however that if we replace $\bar{v}_m \approx \sqrt{\frac{T}{m}}$ we do not get the correct dependence of the resistivity at room temperature. To obtain the correct dependence we need quantum mechanics.
Now the rate at which energy is dissipated by a resistor is

\[ P = \frac{\Delta E}{\Delta t} = \frac{\Delta Q \cdot V}{\Delta t} \]

or

\[ P = IV = I^2R = \frac{V^2}{R} \]