Imagine a plane wave which propagates in an insulator \((\varepsilon > 0)\) and propagates into a conductor \((\varepsilon > 0)\).

For simplicity we will assume EM at normal incidence.

1. What are the boundary conditions?

\[
\begin{align*}
\varepsilon_i \mathbf{E}_i^\perp &= \varepsilon_c \mathbf{E}_c^\perp = \sigma_f \\
\mathbf{B}_i^\parallel &= \mathbf{B}_c^\parallel \\
\frac{\mathbf{B}_i^\perp}{\mu_i} &= \frac{\mathbf{B}_c^\perp}{\mu_c} = \mathbf{k}_t \times \hat{n}
\end{align*}
\]

Note that for Ohmic conductors while there is a non-zero flux, it would take an electric field confined parallel to the surface in a infinitesimally thin strip to create a non-zero \(\mathbf{k}_t\), so we can safely assume \(\mathbf{k}_t = 0\).

Assuming normal incidence \(\mathbf{k}_0 = \mathbf{k}_z \hat{z}\) \(\mathbf{k}_x = -\mathbf{k}_z \hat{z}\) \(\mathbf{k}_y = \mathbf{k}_z \hat{z}\)

\[
\begin{align*}
\mathbf{E}_z &= E_0 z e^{-i(\gamma z - \omega t)} \\
\mathbf{B}_z &= \frac{E_0}{\mu_0} e^{-i(\omega z - \gamma t)} \\
\mathbf{E}_y &= E_0 y e^{-i(\gamma z - \omega t)} \\
\mathbf{B}_y &= \frac{E_0}{\mu_0} e^{-i(\omega z - \gamma t)} \\
\mathbf{E}_x &= E_0 x e^{-i(\gamma z - \omega t)} \\
\mathbf{B}_x &= \frac{E_0}{\mu_0} e^{-i(\omega z - \gamma t)}
\end{align*}
\]
Remember \( \tilde{E} \) is a complex number.

which it is attenuated as it penetrates into the conductor.

Let's now use the boundary conditions

Since \( \mathbf{E}_1^+ = \mathbf{E}_2^- = 0 \) \( \sigma \mu \) must be zero

At \( z = 0 \) \( \tilde{E}_{10} + \tilde{E}_{01} = \tilde{E}_1 = \tilde{E}_{01} = \tilde{E}_2 \)

\( \tilde{E}_{10} + \tilde{E}_{01} = \tilde{E}_{01} \) \( \text{(*)} \)

\( \mathbf{B}_{10}^+ \) are also zero and for the parallel components of \( \mathbf{B} \) we have

\[
\left( \tilde{E}_{01} - \tilde{E}_{00} \right) = \frac{k_0}{\mu V_1} \tilde{E}_{01}
\]

or

\[
\left( \tilde{E}_{10} - \tilde{E}_{01} \right) = \frac{k_0}{\mu V_1} \tilde{E}_{10}
\]

Combining \( \text{(*)} \) and \( \text{(**)} \) we get

\[
\begin{align*}
\frac{\tilde{E}_{10}}{E_{01}} &= \frac{2}{1 + \frac{\mu V_1}{\mu \omega} k_0} \quad \text{Complex Fresnel Equations} \\
\frac{\tilde{E}_{01}}{E_{01}} &= \frac{\left( 1 - \frac{\mu V_1}{\mu \omega} k_0 \right)}{\left( 1 + \frac{\mu V_1}{\mu \omega} k_0 \right)}
\end{align*}
\]

For a good conductor \( \sigma \mu \omega >> 1 \) \( k_0 \to \infty \) so

\( \tilde{E}_{10} \to 0 \) and \( \tilde{E}_{01} \to - \tilde{E}_{01} \) Reflection with 180° phase shift

This is why good conductors make good mirrors!