PHYS 3320

UNIVERSITY OF COLORADO AT BOULDER

PRINCIPLES OF ELECTRICITY AND MAGNETISM II

SPRING, 2010 (AM Rey)

Homework 9

(Due Date: Mon, Apr 5, in class. Recall: late homework will not be accepted)

NOTE: Be sure to show your work and explain what you are doing

1. REFLECTION AND REFRACTION OF EM WAVES WITH PERPENDICULAR POLARIZATION [30 points]

In the lecture and text we have looked at the reflection and transmission of EM plane waves at a plane interface between two media. The one case we have not looked at is waves with polarization perpendicular to the plane of incidence. Let the interface once more be the xy plane (z = 0), and the incident electric wave be

\[ E_I(r, t) = \tilde{E}_0 e^{i(k_I \cdot r - \omega t)} \hat{y}, \]

The propagation vector \( k_I = k_I (\sin \theta_I, 0, \cos \theta_I) \). Assume medium 1 has permittivity, permeability, and index of refraction \( \epsilon_1, \mu_1, \) and \( n_1 \); those of medium 2 are \( \epsilon_2, \mu_2, \) and \( n_2 \).

(a) Find the Fresnel equations for \( \tilde{E}_{0R}/\tilde{E}_{0I} \) and \( \tilde{E}_{0T}/\tilde{E}_{0I} \), and sketch \( \tilde{E}_{0R}/\tilde{E}_{0I} \) and \( \tilde{E}_{0T}/\tilde{E}_{0I} \) for the case that \( \mu_1 n_2/\mu_2 n_1 = 1.5 \).

(b) Confirm that your Fresnel equations reduce to the correct forms at normal incidence.

(c) Compute the reflection and transmission coefficients R and T, and check that they add up to 1.

(d) For this polarization case and with distinguishable media (\( \epsilon, \mu, \) and \( n \) are different for the two media), is there a Brewster’s angle where the reflected amplitude is zero? If so, find an expression for the angle in terms of the media properties (e.g., \( n_1, n_2 \)).
2. TOTAL INTERNAL REFLECTION [30 points] According with Snell’s law when light passes from an optically dense medium into a less dense one ($n_1 > n_2$) the propagation vector bends away from the normal (see Fig 1).

In particular if the light is incident at a critical angle

$$\theta_c = \sin^{-1}(n_2/n_1)$$  \hspace{1cm} (2)

the $\theta_T = 90^\circ$ and the transmitted ray just grazes the surface. If $\theta_I$ exceeds $\theta_c$ there is no refracted ray at all, only a reflected one. This is the phenomenon of total internal reflection on which light pipes and fiber optics are based. But the fields are not completely zero at medium; what one gets is a so called evanescent waves, which is rapidly attenuated and transports no energy into medium 2.

Let’s construct an evanescent wave by quoting the results derived in class with $k_T = \omega n_2/c$ and $k_T = k_T(\sin \theta_T, 0, \cos \theta_T)$ but with $\sin \theta_T = n_1/n_2 \sin \theta_I$ greater than one and $\cot \theta_T = \sqrt{1 - \sin^2 \theta_T} = i \sqrt{\sin^2 \theta_T - 1}$ is imaginary. Show that

(a) $E_T(r,t) = \tilde{E}_0 T e^{i(kx - \omega t)} e^{-\kappa z}$ where $\kappa = \omega/c \sqrt{n_1^2 \sin^2 \theta_I - n_2^2}$ and $k = (\omega n_1/c) \sin \theta_I$. Interpret your answer.

(b) Calculate the reflection coefficient for polarization parallel to the plane of incidence.

Let’s now apply it to a real case of an optical fiber shown in Fig.2 The index of refraction of the inner core is 1.480 , and the index of refraction of the outer cladding is 1.44.

(c) What is the critical angle for the core-cladding interface?

(d) For what range of angles in the core at the entrance of the fiber ($\theta_2$) will the light be completely internally reflected at the core-cladding interface?

(e) What range of incidence angles in air does this correspond to?

(f) If light is totally internally reflected at the upper edge of the fiber, will it necessarily be totally internally reflected at the lower edge of the fiber (assuming edges are parallel)?
3. LIGHT FROM AN AQUARIUM [30 points]

Light of angular frequency $\omega$ passes from water (medium 0, $n_0 = 4/3$), through a slab of glass of thickness $d$ (medium 1 $n_1 = 3/2$), and into air (medium 2, $n_2 = 1$). Assuming all $\mu$ are equal

(a) Show that the transmission coefficient for normal incidence is:

$$T = \frac{4n_0n_2}{(n_0 + n_2)^2 + \left(\frac{n_0^2 - n_2^2}{n_1^2}\right)\sin^2 \left(\frac{n_1\omega d}{c}\right)}^{-1}$$  \hspace{1cm} (3)

(b) How well can you see the fish? How well can it see you?