NOTE: Be sure to show your work and explain what you are doing

1. RLC SERIES CIRCUIT [30 points]

Consider the RLC series circuit shown in the figure; first we’ll use a battery to supply EMF, then an AC power supply.

(a) Consider the DC circuit. If the switch is open, but then closed at time $t = 0$, what is the current through the capacitor as a function of time? Also, what is the voltage across the capacitor as a function of time?

(b) Now replace the battery with an ideal AC power supply that provides a voltage $V_0 \sin \omega t$. After letting all the transients from turning on the power die away, what is the current through the capacitor as a function of time? Make a sketch of the magnitude of the current as a function of frequency.

2. Force on permeable rod in solenoid [30 points]

A long cylindrical rod (permeability $\mu$, length $L_0$, radius $R$, $R \ll L_0$) is partially inserted into a long solenoid (length $L_1$, radius $R$, $R \ll L_1$, turn density $n$, fixed current $I_0$).

(a) How large is the force on the rod?

(b) Is the rod pulled into or pushed out of the solenoid? Does it matter whether the material is paramagnetic or diamagnetic?

(Hint: Start by considering how much energy is contained in the magnetic fields.)
3. DISPLACEMENT CURRENT IN COAX [30 points]

Consider a coax cable as an "infinite" length wire of radius a surrounded by a thin conducting cylinder, coaxial with the wire, with inner radius b and outer radius c. Again, assume \( a \ll b \) and \( c - b \ll b \) (thin shell and wire), as show in the figure.

(a) Calculate the self inductance per length.

(b) Calculate the induced \( E \) field for a particular time dependent current \( I(t) = I_0 \cos \omega t \), which flows along the wire and a corresponding current \( I(t) \) flows in the opposite direction on the outer cylinder. Assume that we have the quasi-static situation in which the currents are identical in magnitude at each moment in time, and the changes in current are sufficiently slow. Find \( E(s,t) \), where \( s \) is the usual radial coordinate and the current on the wire is \( I_0 \) in the +z direction at \( t = 0 \). You can assume that the magnitude of \( E \to 0 \) as \( s \to \infty \). Hint: If you think about the analogy between current density and magnetic flux, what kind of amperian loop should be useful?

(c) Find the displacement current density \( J_d \) for this \( E \), and integrate it to get the total displacement current \( I_d \). Compare the magnitudes of \( I_d \) and \( I_0 \), and, in particular, determine at which frequency one would have \( I_d \) equal to 5% of \( I_0 \) assuming \( b = 4 \) mm and \( a = 0.1 \) mm.
4. ENERGY FLOW IN COAX [30 points]

We now want to investigate energy flow in the same cylindrical coax cable defined in question 3. However, we will first only look at fields constant in time, not varying in time. Assume that constant current $I$ flows in the $+z$ direction on the inner wire and that total current $I$ flows in the opposite direction in the shell. Also assume that there is a constant voltage difference $V$ between the wire and the shell, as shown.

(a) Find the capacitance per length of the coax. Multiply the self inductance per length found in 3a) by the capacitance per length. What do you get? Can you give a physical interpretation of your answer?

(b) Now for a steady current and voltage case: Find $E$ and $B$ everywhere in space. You may assume that the coax cable (wire plus shell) is neutral.

(c) Calculate the Poynting vector $\vec{S} = (\vec{E} \times \vec{B})/\mu_0$. The magnitude of $S$ gives the energy flux density and represents the power per area moving through space. Does its direction make sense for the coax? Integrate this flux through the cross sectional area of the coax to find the power transported down the coax line. Does your answer make sense relative to the circuit maintaining the current and voltage?