Please fill out the following test to the best of your ability (closed book and by yourself). This will not count towards your final grade in the course. Do your best to get to all the questions on the test but do NOT take more than 60 min to do it.

Name: _______________________  ID: ______________________

1- What is the magnitude $|z|$ of the following complex number $|z|: z=3-4i$?

1b- Did you learn about complex numbers in a previous course?  Yes  No

1c- Was problem 1 difficult?  1- Very hard  2- Somewhat hard  3- Avg  4- Somewhat easy  5- Very easy

2- What is the real part of the expression $re^{i\theta}$ (where $r$ and $\theta$ are real constants)?

2b- Was problem 2 difficult?  1- Very hard  2- Somewhat hard  3- Avg  4- Somewhat easy  5- Very easy

3- Which of the following is a possible solution to $5 \frac{d^2x}{dt^2} + 4 \frac{dx}{dt} + x = 0$

(a) $Ae^{2t/5} + Be^{-2t/5}$

(b) $Ae^{-2t/5} \cos \left(\frac{t}{5}\right)$
(c) $A \cos \left( \frac{t}{5} \right) + B \sin \left( \frac{t}{5} \right)$

(d) $A e^{-2t/5} + B t e^{-2t/5}$

(e) $A \cosh \left( \frac{t}{5} \right) + B \sinh \left( \frac{t}{5} \right)$

3b- Was problem 3 difficult?  
1- Very hard  
2- Somewhat hard  
3- Avg  
4- Somewhat easy  
5- Very easy

4- What is the magnitude $|z|$ of the following complex number: $z=5ie^{\pi/3}$?

4b- Was problem 4 difficult?  
1- Very hard  
2- Somewhat hard  
3- Avg  
4- Somewhat easy  
5- Very easy

5- What is the solution to $\int_0^{4\pi} 3e^{i(x+\pi/2)} \, dx$

5b- Did you learn about this in a previous course?  
Yes  
No

5c- Was problem 5 difficult?  
1- Very hard  
2- Somewhat hard  
3- Avg  
4- Somewhat easy  
5- Very easy

6- A new grocery store just opened in the middle of town. In order to attract costumers, they hold an original promotion where each costumer can save money on their purchase by rolling a traditional six face die -- the higher the outcome, the larger the savings. For instance, if the costumer rolls a “6”, he (or she) saves 6% of the total purchase amount, a roll of “5” leads to a saving of 5%, and so forth. What is average saving in percent (in %) they should expect to pay out?
6c- Was problem 6 difficult?  1- Very hard  2- Somewhat hard  3- Avg  4- Somewhat easy  5- Very easy

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7- Match the real part of the function $g(x)$ (denoted as $(\text{Re}[g(x)])$ to the real part of its Fourier Transform, denoted as $\text{Re}(\tilde{f}(k_x))$.

7b- Did you learn about Fourier transforms in a previous course?  Yes  No

If YES, which course(s)?

7c- Was problem 7 difficult?  1- Very hard  2- Somewhat hard  3- Avg  4- Somewhat easy  5- Very easy

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8- Can you approximate $f(x) = (1 + x)^7$ as a linear expression of the form $f(x) \approx a + bx$ (assuming $x \ll 1$)? If you can, what are the values for $a$ and $b$: 
8b- Did you learn about this in a previous course? Yes  No

8c- Was problem 9 difficult?  1- Very hard  2- Somewhat hard  3- Avg  4- Somewhat easy  5- Very easy

9- Evaluate \[ \frac{\partial}{\partial x} \left( \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{(x^2+y^2)}{2\sigma^2}} \right) \] (\(\sigma\) is a constant).

9b- Was Q10 difficult?  1- Very hard  2- Somewhat hard  3- Avg  4- Somewhat easy  5- Very easy

10- Which Fourier series is the correct expansion for the function \(f(x)\)?

(a) \(f(x) = 1 + \frac{4}{\pi} \sin \pi x + \frac{4}{3\pi} \cos \pi x + \frac{4}{5\pi} \sin 3\pi x + \cdots\)

(b) \(f(x) = \frac{4}{\pi} \sin \pi x + \frac{4}{3\pi} \sin 3\pi x + \frac{4}{5\pi} \sin 5\pi x + \frac{4}{7\pi} \sin 7\pi x + \cdots\)

(c) \(f(x) = 1 + \frac{4}{\pi} \sin \pi x + \frac{4}{3\pi} \sin 3\pi x + \frac{4}{5\pi} \sin 5\pi x + \cdots\)

(d) \(f(x) = \frac{4}{\pi} \sin \pi x + \frac{4}{3\pi} \cos \pi x + \frac{4}{3\pi} \sin 3\pi x + \frac{4}{3\pi} \cos 3\pi x + \cdots\)

(e) \(f(x) = 1 + \frac{4}{\pi} \cos \pi x + \frac{4}{3\pi} \cos 3\pi x + \frac{4}{5\pi} \cos 5\pi x + \cdots\)
10b- Did you learn about Fourier series in a previous course? Yes  No

10c- Was Q11 difficult?  1- Very hard  2- Somewhat hard  3- Avg  4- Somewhat easy  5- Very easy

11- What is the shortest distance $x$ over which the function $f(x) = \cos\left(\frac{\pi x}{2p} - \frac{\pi}{3p}\right)$ repeats itself (i.e. period).

(a) $\frac{p}{2}$  (b) $p$  (c) $2p$

(d) $4p$  (e) $\frac{p}{\pi}$  (f) $\frac{p}{2\pi}$

11b- Was Q12 difficult?  1- Very hard  2- Somewhat hard  3- Avg  4- Somewhat easy  5- Very easy

12- How many linearly independent solutions (i.e. $y_1(t)$, $y_2(t)$, $y_3(t)$, $y_4(t)$, ...) does $5 \frac{d^3y}{dt^3} - \frac{d^2y}{dt^2} + y = 0$ have?

(a) It depends on the initial conditions: $y(0), y'(0)$...

(b) 2

(c) 4

(d) 1

(e) 3

12b- Did you learn about this in a previous course? Yes  No

12c- Was Q13 difficult?  1- Very hard  2- Somewhat hard  3- Avg  4- Somewhat easy  5- Very easy

13- Two traveling waves are described by the equations: $Y_1(x,t) = 8\cos(4x + 2t)$ and $Y_2(x,t) = 2\cos(2x + t)$. Which wave has the highest propagation speed?

(a) Wave 1

(b) Wave 2

(c) Both waves have the same speed
14- Given the solution to a linear differential equation \( y(x) = A\sin(kx) + B\cos(kx) \), the boundary condition \( y'(0) = 0 \), implies:

(a) \( A = 0 \)
(b) \( B = 0 \)
(c) \( k = 0 \)
(d) \( k = n\pi, \ n = 1,2,3 \ldots \)

(e) A trivial solution for all \( x \) (i.e. \( y(x) = 0 \))

15- The velocity of a particle is given by \( \vec{v}(t) = \sin(t) \hat{i} + \cos(t) \hat{j} \). Express the velocity \( \vec{v}(t) \) in terms of the unit vectors \( \hat{e}_1 \) and \( \hat{e}_2 \):

\[ \hat{e}_2 \]
\[ 45^\circ \]
\[ \hat{e}_1 \]

16- Evaluate \( \int_{z=0}^{z=L} \int_{y=0}^{y=1/\sqrt{z}} \int_{x=0}^{x=y} 2z \, dx \, dy \, dz \):
16b- Did you learn about this in a previous course? Yes No

16c- Was Q16 difficult? 1- Very hard 2- Somewhat hard 3- Avg 4- Somewhat easy 5- Very easy

17- Large forest fires (or any large fire for that matter) are known to produce hurricane force winds in their vicinity. Large temperature gradients drive air masses at spectacular speeds. What mathematical expression is appropriate to describe the spatial and temporal variation of the density of air (kg/m^3) in and around the fire?

(a) A scalar field \( \rho = \rho(x(t), y(t), z(t)) \)

(b) A vector field \( \vec{\rho} = (\rho_x(x(t), y(t), z(t)), \rho_y(x(t), y(t), z(t)), \rho_z(x(t), y(t), z(t))) \)

(c) In this particular case, the density \( \rho \) cannot be expressed as a scalar or a vector field.

Please comment on your choice:

17b- Did you learn about this in a previous course? Yes No

17c- Was Q17 difficult? 1- Very hard 2- Somewhat hard 3- Avg 4- Somewhat easy 5- Very easy

18- The matrix \( R = \begin{pmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{pmatrix} \) takes a vector \( r = (r_x, r_y, r_z) \) and rotates it by an angle \( \theta \) about an axis. What is the axis of rotation?

(a) x-axis -- clockwise  (b) x-axis -- counterclockwise

(c) y-axis -- clockwise  (d) y-axis -- counterclockwise

(e) z-axis -- clockwise  (f) z-axis -- counterclockwise

18b- Did you learn about this in a previous course? Yes No
19- The vector \( \mathbf{v} = (v_x, v_y, v_z) \) is an eigenvector of the matrix
\[
R = \begin{pmatrix}
\cos(\theta) & 0 & \sin(\theta) \\
0 & 1 & 0 \\
-\sin(\theta) & 0 & \cos(\theta)
\end{pmatrix}
\]
(same rotation matrix as in Prob. 21).

In other words, \( \mathbf{v} = (v_x, v_y, v_z) \) satisfies \( R\mathbf{v} = \lambda\mathbf{v} \), where \( \lambda \) is the eigenvalue associated with the eigenvector \( \mathbf{v} \). Which of the vector below is a candidate for \( \mathbf{v} \)?

(a) \( \mathbf{v} = (0,0,-1) \)  
(b) \( \mathbf{v} = (-1,0,1) \)

(c) \( \mathbf{v} = (1,1,1) \)  
(d) \( \mathbf{v} = (0,-1,0) \)

(e) \( \mathbf{v} = (1,0,-1) \)  
(f) \( \mathbf{v} = (-1,0,0) \)

19b- Did you learn about this in a previous course? Yes  No

19c- Was Q2 difficult?  
1- Very hard  2- Somewhat hard  3- Avg  4- Somewhat easy  5- Very easy

20- The function \( f(x) \), shown below, is Taylor expanded to second order in \( x \) about \( x = 0 \):
\[
f(x) \sim a_0 + a_1 x + a_2 x^2.
\]
What can you say about the signs of \( a_0 \), \( a_1 \), and \( a_2 \)?

(a) \( a_0 > 0, a_1 > 0, a_2 > 0 \)  
(b) \( a_0 < 0, a_1 < 0, a_2 < 0 \)

(c) \( a_0 > 0, a_1 < 0, a_2 < 0 \)  
(d) \( a_0 < 0, a_1 > 0, a_2 < 0 \)

(e) \( a_0 < 0, a_1 < 0, a_2 > 0 \)
**20b**- Did you learn about this in a previous course?  Yes  No

**20c**- Was Q23 difficult?  1- Very hard  2- Somewhat hard  3- Avg  4- Somewhat easy  5- Very easy

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**21**- The plot below represents the vector field $\vec{F} = (F_x(x,y), F_y(x,y))$, where the length of the arrow is proportional to the field magnitude at that point. What can you say about the divergence of the vector field ($\nabla \cdot \vec{F}$) at the center of the target sign in the lower left corner?

(a) $\nabla \cdot \vec{F} = 0$

(b) $\nabla \cdot \vec{F} > 0$

(c) $\nabla \cdot \vec{F} < 0$

(d) There is not enough information

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**21b**- Did you learn about this in a previous course?  Yes  No
21c- Was Q24 difficult?  1- Very hard  2- Somewhat hard  3- Avg  4- Somewhat easy  5- Very easy

22- The plot below represents a two-dimensional vector field $\vec{B} = (B_x(x,y), B_y(x,y))$, where the length of the arrow is proportional to the field magnitude at that point. What can you say about the z-component of $\nabla \times \vec{B}$ at the center of the target sign in the lower left corner? (+z is out of the page)

(a) The z-component of $\nabla \times \vec{B}$ is zero
(b) The z-component of $\nabla \times \vec{B}$ is greater than zero
(c) The z-component of $\nabla \times \vec{B}$ is smaller than zero
(d) There is not enough information

22b- Did you learn about this in a previous course? Yes   No
22c- Was Q25 difficult? 1- Very hard 2- Somewhat hard 3- Avg 4- Somewhat easy 5- Very easy
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23- What is the divergence of the vector field \( \vec{A} = x \hat{x} + y \hat{y} + z \hat{z} \)?

23b- Was Q25 difficult? 1- Very hard 2- Somewhat hard 3- Avg 4- Somewhat easy 5- Very easy
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24- What is the curl of the vector field \( \vec{B} = x \hat{x} - y \hat{y} + \hat{z} \)?

24b- Was Q25 difficult? 1- Very hard 2- Somewhat hard 3- Avg 4- Somewhat easy 5- Very easy
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How seriously did you just take this diagnostic exam?

(a) I pretty much blew it off, didn't think much about a lot of the answers.
(b) I took it sort of seriously, but when I didn't know an answer I didn't think very hard about it.
(c) I took it seriously, and thought about my answers.

If you imagine getting a letter grade on this test, what do you think that grade would be? _____