P3320 LECTURE 43

Relativistic treatment of the potentials.

What is E_i in terms of potentials?

$$\vec{E} = -\operatorname{grad}\Phi - \frac{1}{c}\frac{\partial \vec{A}}{\partial t}$$

so $$E_i = -\left(\frac{\partial \Phi}{\partial x_i} + \frac{1}{c}\frac{\partial A_i}{\partial t}\right)$$

$$\vec{B} = \operatorname{curl}\vec{A}$$

so $$B_i = \frac{\partial}{\partial_{i+1}} A_{i+2} - \frac{\partial}{\partial_{i+2}} A_{i+1}$$

Consider these as components of $F^{\mu\nu}$:

$E_i = F^{0i}$, so $\quad F^{0i} = -\left(\frac{\partial \Phi}{\partial x_i} + \frac{\partial A_i}{\partial x^0}\right)$ (remember $x^0 = ct$)

$B_1 = F^{23}$: $\quad F^{23} = \frac{\partial A_3}{\partial x_2} - \frac{\partial A_2}{\partial x_3}$

$B_2 = -F^{13}$: $\quad -F^{13} = \frac{\partial}{\partial x_3} A_1 - \frac{\partial}{\partial x_1} A_{x_3} \Rightarrow F^{13} = \frac{\partial A_3}{\partial x_1} - \frac{\partial A_1}{\partial x_3}$

$B_3 = F^{12}$: $\quad F^{12} = \frac{\partial A_2}{\partial x_1} - \frac{\partial A_1}{\partial x_2}$

So for the F^{ij} $(i,j \neq 0)$ components, can write

$$F^{ij} = \frac{\partial A_j}{\partial x_i} - \frac{\partial A_i}{\partial x_j} \quad \Big\} \text{ i,j go 1,2,3 \underline{only}!}$$

V/c in SI units

For F^{0i} we can write something very similar if we set $A_0 \equiv \Phi$:

$$F^{0i} = -\frac{\partial A_i}{\partial x^0} - \frac{\partial A_0}{\partial x_i} = +\frac{\partial A_i}{\partial x_0} - \frac{\partial A_0}{\partial x_i}$$

So now, using $x_0 = -x^0$, can write entire tensor as $F^{\mu\nu} = \frac{\partial A_\nu}{\partial x_\mu} - \frac{\partial A_\mu}{\partial x_\nu}$

$$\boxed{F^{\mu\nu} = \partial^\mu A_\nu - \partial^\nu A_\mu}$$

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→ This is an alternative (more elegant) way to derive the field tensor.

Other delightful things that can come from this description:

Recall our two gauge conditions:

Coulomb: $\operatorname{div}\vec{A} = 0$

Lorentz: $\operatorname{div}\vec{A} = -\frac{1}{c}\frac{\partial \Phi}{\partial t}$

looks like: $\partial^i A_i = -\frac{1}{c}\frac{\partial A_0}{\partial t} = -\frac{\partial A_0}{\partial x^0}$

$$\Rightarrow \partial^\mu A_\mu = 0 \Leftarrow \text{4-dimensional divergence vanishes!}$$

In Lorentz gauge, recall the equations of motion of the potentials:

$$\nabla^2 \Phi - \frac{1}{c^2}\frac{\partial^2 \Phi}{\partial t} = -4\pi\rho$$

$$\nabla^2 \vec{A} - \frac{1}{c^2}\frac{\partial^2 \vec{A}}{\partial t^2} = -\frac{4\pi}{c}\vec{J}$$

or $\frac{\partial}{\partial x_\nu}\frac{\partial}{\partial x^\nu}A^\mu = -\frac{1}{c}J^\mu$ where $\frac{\partial}{\partial x_\nu}\frac{\partial}{\partial x^\nu} \equiv \Box^2$.