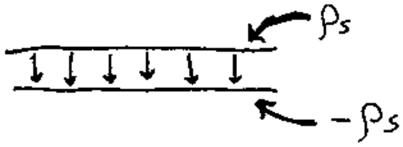
P3320 LECTURE 40

Relativistic electromagnetism: How do the fields transform?

Start with $\vec{E}$: parallel plates work well to demonstrate:

$\vec{E} = -4\pi\rho_s \vec{e}_y$ 

$\longrightarrow$ boost in $\vec{e}_x$

Boosted frame: charged plates carry same charge Q as before, but are shorter by factor of $\frac{1}{\gamma}$ so $\rho_s' = \gamma\rho_s$, and

$E_y' = -4\pi\rho_s\gamma\,\vec{e}_y = \gamma E_y$

So $\vec{E}$ components <u>normal</u> to boost direction get increased by γ.

For parallel component, consider boost in y. $\rho_s' = \rho_s$ now, so $E_{\parallel}' = E_{\parallel}$. (the plates are closer together, though.)

What is $\vec{E}$ for a charge in uniform motion? Put the charge at rest in the primed frame, at origin of both at $t=0$.

Since $E'_{\parallel} = E_{\parallel}$ and $E'_{\perp} = \gamma E_{\perp}$, and $\vec{E}' = \frac{Q}{r'^2}\vec{e}_{r'} = \frac{Q\vec{r}'}{r'^3}$

so $E'_{x'} = \frac{Qx'}{r'^3} = \frac{Qx'}{(x'^2+y'^2+z'^2)^{\frac{3}{2}}}$ etc. for $E'_{y'}, E'_{z'}$.

Now, $\vec{E}$ (in unprimed frame) is, still as fn. of primed coordinates:

$$E_x(\vec{r}') = \frac{Qx'}{(x'^2+y'^2+z'^2)^{\frac{3}{2}}}$$

$$E_y(\vec{r}') = \gamma\frac{Qy'}{(x'^2+y'^2+z'^2)^{\frac{3}{2}}}$$

$$E_z(\vec{r}') = \gamma\frac{Qz}{(x'^2+y'^2+z'^2)^{\frac{3}{2}}}$$

To go to unprimed coordinates, boost the coordinates:

$$x' = \gamma(x+\beta ct), \quad y' = y, \quad z' < z$$

Evaluate fields at $t=0$.

$$\left.\begin{aligned} E_x(\vec{r}) &= \frac{Q\gamma x}{(\gamma^2x^2+y^2+z^2)^{\frac{3}{2}}} \\ E_y(\vec{r}) &= \frac{Q\gamma y}{(\gamma^2x^2+y^2+z^2)^{\frac{3}{2}}} \\ E_z(\vec{r}) &= \frac{Q\gamma z}{(\gamma^2x^2+y^2+z^2)^{\frac{3}{2}}} \end{aligned}\right\} \vec{E}(\vec{r}) = \frac{Q\gamma\vec{r}}{(\gamma^2x^2+y^2+z^2)^{\frac{3}{2}}}$$

We can compare this to the formula we got using retarded potentials by calling the boost direction z so $\cos\theta = \frac{z}{r}$:

$$\vec{E}(\vec{r}) = \frac{Q\gamma\vec{r}}{(\gamma^2 r^2\cos^2\theta + r^2\sin^2\theta)^{\frac{3}{2}}} = \frac{Q\gamma\vec{r}}{[(\gamma^2-1)\cos^2\theta + 1]r^3}$$

$$= \frac{Q(1-\beta^2)\vec{r}}{(1-\beta^2\sin^2\theta)^{\frac{3}{2}}\, r^3}$$ as we got before.