Special relativity – from an E&M perspective.

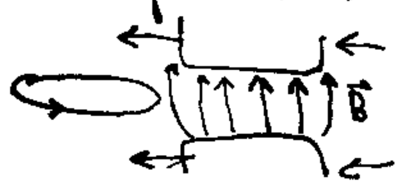
SR comes from two basic postulates:

1) The laws of physics are the same in all inertial reference frames, and
2) The (vacuum) speed of light is the same in all reference frames, independent of any motion of the source of radiation.

On their surface the Biot-Savart and Maxwell equations do not seem to satisfy #1 above: $\vec{F} = q\frac{\vec{V}}{c}\times\vec{B}$: the velocity appears explicitly in the formula!

But, it turns out that the forces (not fields) calculated in many dynamics problems are actually independent of the frame in the Maxwell eqn. Classic example is a loop EMF when brought through a magnetic field:

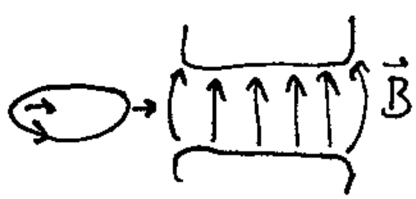
Wire frame:



Magnet moving at $-\vec{v}$, wire loop stationary

Magnetic force is zero, but since $\frac{d\vec{B}}{dt} \neq 0$ there is an induced electric field: $\text{curl}\,\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t}$. By Stokes's law, $\int_{loop} d\vec{l}\cdot\vec{E} = \mathcal{E} = -\frac{1}{c}\frac{\partial\Phi_m}{\partial t}$.

Magnet frame:



Magnet stationary, wire loop moving at $+\vec{V}$.

Now, since the charges in the wire are moving, there is a <u>magnetic</u> force on them. If $\vec{B}$ is uniform over the loop, this force cancels when integrated: $\vec{F} = \rho_\ell \int_{loop} d\ell \cdot \left(\frac{\vec{v}}{c} \times \vec{B}\right)$. But if the "motional EMF" is calculated in terms of the instantaneous magnetic flux through the wire $\Phi_m(t)$, we find that $\mathcal{E} = -\frac{1}{c}\frac{\partial \Phi_m}{\partial t}$ $\longrightarrow$ same value, though here the force on the charge is magnetic, and analyzing in the other reference frame it was electric.

Now, consider how we transform systems from one frame to another: In the "magnet frame", let $\vec{r}$ be the position of the center of the wire loop. Clearly, $\vec{r} = \vec{r}_0 + \vec{v}t$. In the "wire frame", the loop position is just $\vec{r}(t) = \vec{r}_0$.

$\Longrightarrow$ the frames are instantaneously coincident in space at $t=0$. Let the coordinates in the loop frame be x', y', z', t' and the coordinates in the magnet frame be x, y, z, t:

Galilean transformations
$$\begin{cases} x' = x \\ y' = y \\ z' = z - vt \\ t' = t \quad \text{(duh!)} \end{cases}$$

(we take the z-axis along $\vec{v}$)

Clearly Newton's second law is invariant, since $a = \frac{d^2\vec{r}}{dt^2}$ and $\frac{d^2\vec{r}'}{dt'^2} = \frac{d^2\vec{r}}{dt^2}$. But Maxwell's equations are <u>not</u> invariant under Galilean transforms.

Lorentz transformations:

$$\left.\begin{aligned} x' &= x \\ y' &= y \\ z' &= \gamma(z - vt) \\ t' &= \gamma\left(t - \frac{v}{c^2}z\right) \end{aligned}\right\}$$

Note that the origins of coord systems coincide when $t = 0$.

You can check (next homework!) that this transformation preserves the speed of light in the boosted frame.

We can combine the x, y, z, t coordinates into a four-vector: unfortunately two ways to do this:

H&M: (x, y, z, ict) so $x \cdot x \equiv x^2 + y^2 + z^2 - c^2t^2$

Comm: (ct, x, y, z) where multiplication metric

contravariant Index: $x^0 = -x_0$ covariant index

$x^1 = x_1$, $x^2 = x_2$, $x^3 = x_3$.

Einstein summation index: $x^\mu x_\mu$ (repeated Greek index)

implies $\sum_{\mu=0}^{3} x^\mu x_\mu$.