P3320 LECTURE 33

Electric dipole radiation: start with vector potential

$$\text{curl } \vec{A} = \frac{1}{c} \text{curl} \int d^3r' \frac{\vec{J}(\vec{r}', t_{ret})}{\mathscr{r}}$$

Since integral over primed coords, move curl inside:

$$\vec{B} = \frac{1}{c} \int d^3r' \; \text{curl} \frac{\vec{J}(\vec{r}', t_{ret})}{\mathscr{r}}$$

This goes as $\frac{1}{\mathscr{r}^2}$ → not radiation field

$$\text{where} \quad \text{curl} \frac{\vec{J}(\vec{r}', t_{ret})}{\mathscr{r}} = \frac{1}{\mathscr{r}} \text{curl} \vec{J}(\vec{r}', t_{ret}) - \vec{J}(\vec{r}', t_{ret}) \times \text{grad} \frac{1}{\mathscr{r}}$$

Note – curl, grad on <u>unprimed</u> coords – so only affect $\vec{J}$ via t_{ret}.

$$\vec{B}_{rad} = \frac{1}{c} \int d^3r' \frac{\text{curl} \vec{J}(\vec{r}', t_{ret})}{\mathscr{r}}$$

$$\text{and} \quad \text{curl} \vec{J} = -\frac{\partial \vec{J}(\vec{r}, t_{ret})}{\partial t} \times \text{grad } t_{ret}$$

$$\text{where} \quad \text{grad } t_{ret} = \underbrace{\text{grad} \, t}_{0} - \frac{1}{c} \text{grad} \, \mathscr{r} = -\frac{1}{c} \vec{e}_{\mathscr{r}}$$

$$\Rightarrow \vec{B}_{rad} = \frac{-1}{c^2} \int d^3r' \frac{1}{\mathscr{r}} \left(-\frac{\partial \vec{J}(\vec{r}', t_{ret})}{\partial t} \times \vec{e}_{\mathscr{r}} \right)$$

$$= \frac{+1}{c^2} \frac{\partial}{\partial t} \left(\int d^3r' \; \frac{\vec{J}(\vec{r}', t_{ret})}{\mathscr{r}} \times e_{\mathscr{r}} \right)$$

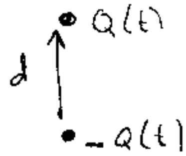
reverse cross product order:

$$= -\frac{1}{c} \vec{e}_{\mathscr{r}} \times \frac{\partial \vec{A}}{\partial t}$$

since we know from Lienhard-Wiechert that $\vec{E}_{rad} = \vec{B}_{rad} \times \vec{e}_{\mathscr{r}}$,

$$\vec{S}_{rad} = \frac{c}{4\pi} \left(\vec{B}_{rad} \times \vec{e}_{\mathscr{r}} \right) \times \vec{B}_{rad} = \frac{c}{4\pi} B_{rad}^2 \, \vec{e}_{\mathscr{r}} = \frac{\vec{e}_{\mathscr{r}}}{4\pi c} \left| e_{\mathscr{r}} \times \frac{\partial \vec{A}}{\partial t} \right|^2$$

Now go back to our two-charge dipole:



$$I = \frac{dQ}{dt} \quad \text{so} \quad \vec{A} = \frac{1}{c}\int_{-\frac{d}{2}}^{\frac{d}{2}} dz' \, \frac{I(t_{ret})}{\mathscr{r}} \, \vec{e}_z$$

Assuming retardation is negligible <u>within</u> the dipole, $t_{ret} = t - \frac{r}{c}$ and large r: $\frac{1}{\mathscr{r}} \to \frac{1}{r}$

then $\vec{A} \approx \frac{1}{cr} d \vec{e}_z I$

$$= \frac{1}{c} \frac{\dot{\vec{p}}(t_{ret})}{r}$$

Now, $\vec{B}_{rad} = -\frac{1}{c} \vec{e}_{\mathscr{r}} \times \frac{\partial \vec{A}}{\partial t}$

$$= -\frac{1}{c^2 r} \vec{e}_{\mathscr{r}} \times \ddot{\vec{p}}(t_{ret}) = \frac{\vec{e}_\phi}{c^2 r} |\ddot{p}(t_{ret})| \sin\theta \quad \text{for } \vec{p} \parallel \vec{e}_z$$

and $\vec{E}_{rad} = \vec{B}_{rad} \times \vec{e}_{\mathscr{r}} = \frac{\vec{e}_\theta}{c^2 r} |\ddot{p}(t_{ret})| \sin\theta$

$\Rightarrow$ can find $|\vec{S}_r|$ easily:

Power $\frac{dP}{d\Omega} = \frac{|\ddot{p}(t_{ret})|^2}{4\pi c^3} \sin^2\theta$

and $P = \frac{2 |\ddot{p}(t_{ret})|^2}{3c^3}$

If $\vec{p}$ depends harmonically on time, $|\ddot{p}| = \omega^2 p_0$ and time avg $\langle \ddot{p}^2 \rangle = \frac{\omega^4 p_0^2}{2}$

So power $P = \frac{p_0^2 \omega^4}{3c^3}$

and angular dist. $\frac{dP}{d\Omega} = \frac{p_0^2 \omega^4}{8\pi c^3} \sin^2\theta$ higher frequency $\rightarrow$ more radiation

Standard Larmor formula angular dist.

Radiation resistance: $I(t) : \frac{\dot{p}}{d}$ where d is the charge separation. $\dot{p} = -i\omega p_0 e^{-i\omega t}$

So $I(t) = \left(\frac{-i\omega p_0}{d}\right) e^{-i\omega t}$ → Call this I_0.

Now, can write average power as:

$$\langle P \rangle = \frac{P_0^2 \omega^4}{3c^3} = \frac{I_0^2 d^2 \omega^2}{3c^3}$$

which is the power that must be supplied to the dipole to keep the dipole moment oscillating. Treat as an Ohm's Law problem: $P = I^2 R$

Now, $\langle I_0^2 \rangle = \frac{I_0^2}{2}$ so radiation resistance is

$$R_{rad} = \frac{8\pi^2}{3c}\left(\frac{d}{\lambda}\right)^2$$ in terms of the wavelength $\lambda = \frac{2\pi}{k}$.

Shape of fields — nice discussion on p.301.