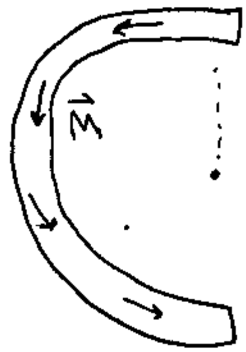
P3320 LECTURE 3 Note — exams Thurs. 10/4, 11/8 ?

Using boundary conditions: fields of a permanent C-magnet.



$\vec{M} \approx M_0 \hat{\phi}$ (not exactly, since this has div $\neq 0$)

Note - this is not a linear material! $\vec{B} \neq \mu \vec{H}$.

No free current anywhere: $\text{curl}\,\vec{H} = 0$

$$\text{div}\,\vec{H} = -4\pi\,\text{div}\,\vec{M}$$

$$\text{div}\,\vec{B} = 0$$

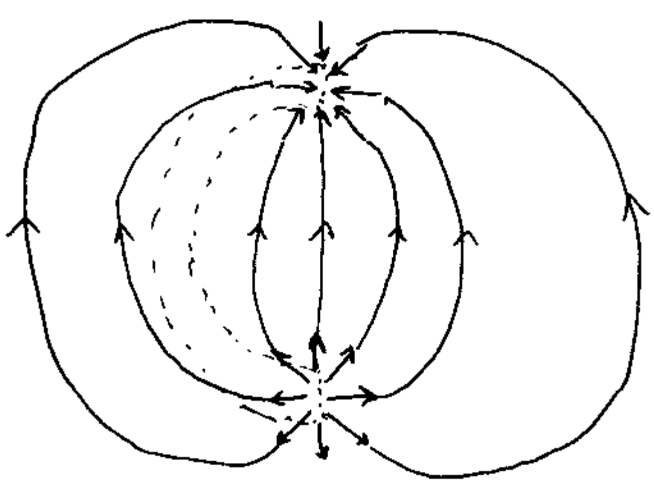
$$\text{curl}\,\vec{B} = \frac{4\pi}{c}\vec{J}_b$$

So at boundaries, $B_\perp$ and $H_\parallel$ will be continuous.

$\Rightarrow$ Sources & sinks of $\vec{H}$ lines are the top & bottom faces, where $\Delta\vec{M} \perp$ surface.

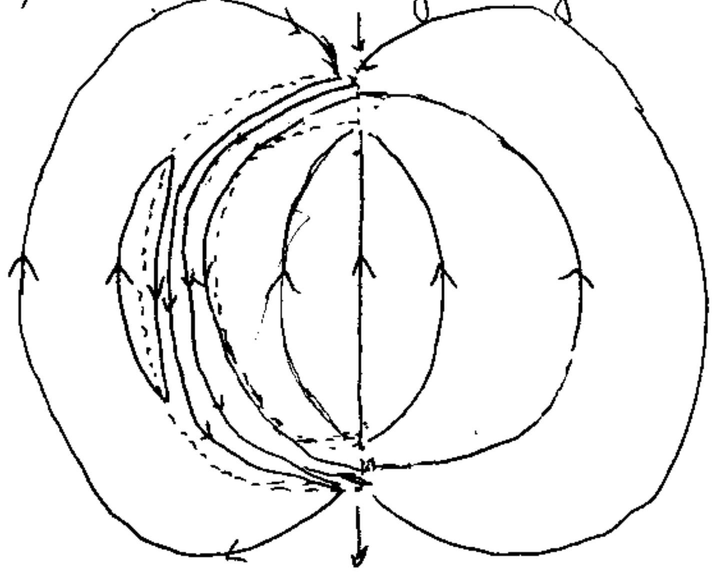
Along curved surface, $\vec{M}_{in} \parallel$ surface and $\vec{M}_{out} = 0$, so div $\vec{M} = 0$ (can also see this because $M_{in\perp} - M_{out\perp} = 0$)

$\Rightarrow$ Along curved surface, div $\vec{H}$ and curl $\vec{H} = 0$, so $H_\parallel$ and $H_\perp$ are continuous! So $\vec{H}$ looks basically like a dipole:



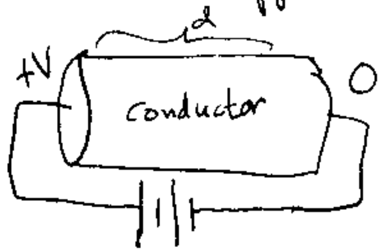
What about $\vec{B}$? Outside material, $\vec{B} = \vec{H}$. Within, $\vec{B} = \vec{H} + 4\pi\vec{M}$, so there's basically a huge extra $\vec{B}$ there:

$\vec{B}$



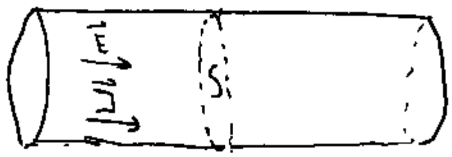
Ohm's Law: We stated early last semester that in electrostatic equilibrium, there is no $\vec{E}$-field in a conductor, since charges would otherwise move around to cancel $\vec{E}$.

But, if we force an $\vec{E}$ field in a conductor (usually by a voltage source, what happens?:



In this case, integrating along the conductor produces a field $E = \frac{V}{d}$. The (empirical) response of the conductor is to have a steady-state current density: $\vec{J} = \sigma\vec{E}$, where σ is the conductivity.

To obtain "engineering" form of Ohm's Law, take a cross-section of the material to find I:



$$I = \int_S d\vec{A} \cdot \vec{J}$$

$V = \int d\vec{l} \cdot \vec{E}$ by definition.

Take a short segment of material, with thickness dl and cross-section $S \perp \vec{J}$, with area A.

For this segment, $dV = d\vec{l} \cdot \vec{E} = d\vec{l} \cdot \frac{1}{\sigma}\vec{J} = \frac{1}{\sigma} J \, dl$
since $d\vec{l} \perp \vec{J}$.

$$I = \int_S d\vec{A} \cdot \vec{J} = AJ \quad \Rightarrow \quad \text{define } dR = \frac{dl}{\sigma A}$$

Now, $dV = I \, dR$

Integrating over length for full object, $V = IR$.