 → EXAM: SAMPLE PROBLEMS UP THIS WEEKEND.

The general case: fields of a moving point charge: two approaches — start with L-W potentials (Griffiths) or start with the Jefimenko fields (the "retarded fields") and take the limit as $\rho(\vec{r}) \to Q\delta(\vec{r}_e - \vec{r})$.

Result is:

$$\vec{E} = Q\left[\frac{(\vec{e}_{\mathcal{R}} - \vec{\beta})(1-\beta^2)}{\mathcal{R}^2(1-\beta_{\mathcal{R}})^3} + \frac{\vec{\mathcal{R}} \times \left((\vec{\mathcal{R}} - \vec{\beta}) \times \vec{a}\right)}{\mathcal{R}^3 c^2 (1-\beta_{\mathcal{R}})^3}\right]_{ret}$$

where $\beta_{\mathcal{R}} \equiv \vec{\beta} \cdot \vec{e}_{\mathcal{R}}$, $\vec{a}$ = acceleration = $\frac{d\vec{u}}{dt}$

and:

$$\vec{B} = \vec{e}_{\mathcal{R}}(t_{ret}) \times \vec{E} \quad .$$

→ $\vec{B}$ is always normal to $\vec{E}$.

Radiation will always be associated with accelerating charges. So to find these fields, look at only field components that depend on $\vec{a}$:

$$\vec{E}_a = Q \frac{\vec{\mathcal{R}} \times \left((\vec{\mathcal{R}} - \vec{\beta}) \times \vec{a}\right)}{c^2(1-\beta_{\mathcal{R}})^3 \mathcal{R}^3}\Bigg|_{t_{ret}}$$

For $\beta \ll 1$, $\quad \vec{E}_a \approx Q \frac{\vec{\mathcal{R}} \times (\vec{\mathcal{R}} \times \vec{a})}{\mathcal{R}^3 c^2}\Bigg|_{t_{ret}}$ (BAC-CAB)

$$= \frac{Q}{c^2 \mathcal{R}^3}\left((\vec{\mathcal{R}} \cdot \vec{a})\vec{\mathcal{R}} - \mathcal{R}^2 \vec{a}\right)\Bigg|_{t_{ret}}$$

and $\vec{B}_a = \frac{\vec{\mathcal{R}} \times \vec{E}_a}{\mathcal{R}}$

Poynting vector becomes another BAC-CAB problem now.

Define θ as angle between $\vec{r}$ and $\vec{a}$:

$$\vec{S}_a = \frac{c}{4\pi} E_a^2 \, \vec{e}_r = \frac{Q^2 a^2 \sin^2\theta}{4\pi c^3 r^2} \vec{e}_r \qquad \Longleftarrow \beta \ll 1 \text{ only!}$$

So radiation is mostly normal to $\vec{a}$, and total power radiated is $P = \int d\Omega \, \frac{S_r}{r^2} = \frac{2e^2a^2}{3c^3}$ (Larmor formula)

The fields on prev. page are relativistically correct (if you treat the retardation properly!) So by not taking $\beta \ll 1$, we can get the radiated power. Two important cases:

- Linear acceleration $(\vec{a} \parallel \vec{\beta} \Rightarrow \vec{a} \times \vec{\beta} = 0)$
- Centripetal acceleration $(\vec{a} \perp \vec{\beta} \Rightarrow \vec{a} \cdot \vec{\beta} = 0)$

Start with linear acceleration:

$$\vec{E}_a = \frac{Q \vec{r} \times (\vec{r} \times \vec{a})}{c^2 r^3 (1 - \vec{\beta} \cdot \vec{e}_r)^3} = \frac{Q}{c^2 r^3 (1 - \vec{\beta} \cdot \vec{e}_r)^3} \left[(\vec{r} \cdot \vec{a}) \vec{r} - r^2 \vec{a} \right]$$

which yields $\vec{S}_a = \dfrac{Q^2 a^2 \sin^2\theta}{4\pi c^3 r^2 (1 - \vec{\beta} \cdot \vec{e}_r)^6} \vec{e}_r$

Note that the integral of $\vec{S} \cdot d\vec{A}$ at a radius R is the flux radiated from the particle at $\underline{t_{ret}} = t - r(t_{ret})/c$

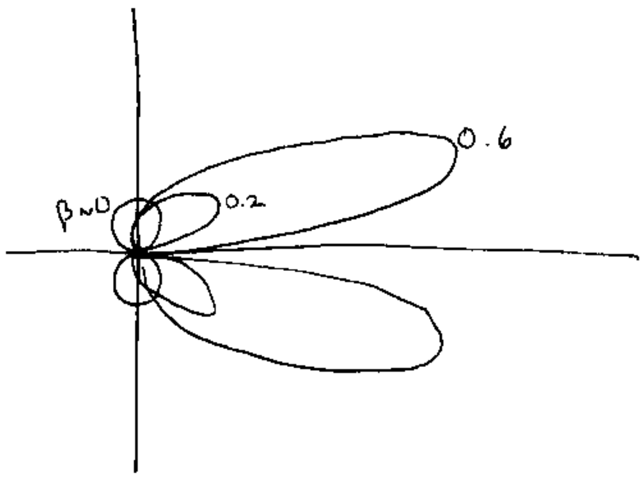
factor $\Rightarrow 1 - \beta\cos\theta = (1 - \vec{\beta} \cdot \vec{r})$.

$\Rightarrow$ only works because $\vec{a} \parallel \vec{\beta}$.

$$\frac{dP}{d\Omega} = \frac{Q^2 a^2 \sin^2\theta}{4\pi c^3 (1 - \beta\cos\theta)^5}$$

$\Rightarrow$ Looks like Larmor at $\beta \ll 1$

$\Rightarrow$ for $\beta \to 1$, huge enhancement at forward angles!



how to read these "lobes"
Draw radial line and then see how far out you cross the lobe.

Circular orbits: synchrotron radiation.

$$\frac{dP}{d\Omega} = \frac{Q^2 a^2}{4\pi c^3} \frac{(1-\beta\cos\theta)^2 - (1-\beta^2)\sin^2\theta\cos^2\phi}{(1-\beta\cos\theta)^5}$$

Weird angular dependence! See p. 283 for diagrams.

Total power is $P = \frac{2Q^2 c}{3\rho^2}\gamma^4$ where ρ is the orbit radius, $\gamma = \frac{1}{\sqrt{1-\beta^2}}$

The γ factor is related to momentum:

$$p = \beta\gamma mc$$

so for a given momentum and radius a lighter particle radiates <u>much</u> more than a heavier particle.