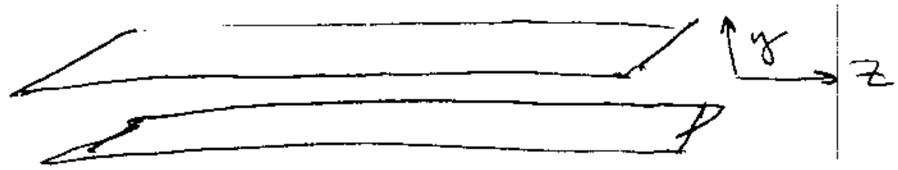
Types of propagating waves: TEM (transverse electric & magnetic field)
TE (trans. electric; $\vec{B}$ has longitudinal component)
TM (transverse magnetic)

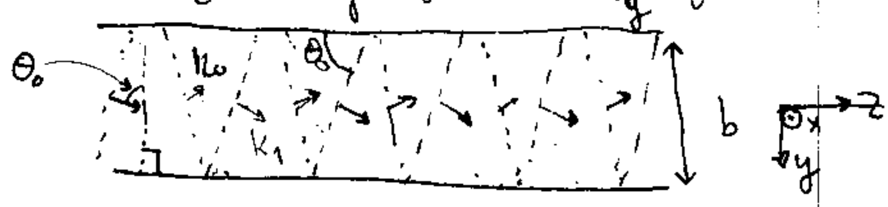
A "bridge" between 2-conductor transmission lines and hollow waveguides: 2 parallel conducting planes



Look at TEM waves between conductors — boundary conditions are that $E_{\parallel}$ and $B_{\perp}$ are continuous. Since $E=B=0$ in conductor, this means $E_{\parallel}=B_{\perp}=0$ at boundary. Therefore:

$$\vec{E}_0 = E_0\vec{e}_x,\ \vec{B}_0 = E_0\vec{e}_y \quad \text{cannot propagate}$$
$$\vec{E}_0 = E_0\vec{e}_y,\ \vec{B}_0 = -E_0\vec{e}_x \quad \text{can propagate at } u=c.$$

Another mode: θ_0 is angle b/w $\vec{k}$ and $\vec{e}_y$ (or between wavefronts and $\vec{e}_z$)



Simultaneous waves with wave vectors $\pm k_y\vec{e}_y + k_z\vec{e}_z$, where each is the reflection of the other. Assume $\vec{E}_0$ is in $\vec{e}_x$: then $\vec{B}_0$ has a z component $\Rightarrow$ this is a TE wave:
Boundary cond. requires $E_{1x} = -E_{0x}$, $B_{0y} = -B_{1y}$, since they must add to zero.

At $y=0$:

So if $\vec{E}_0 = \vec{e}_x E_{00} e^{i(\vec{k}_0\cdot\vec{r}-\omega t)}$ where $\vec{k}_0\cdot\vec{r} = (-y\cos\theta_0 + z\sin\theta_0)k_0$

then $\vec{E}_1 = -\vec{e}_x E_{00} e^{i(\vec{k}_1\cdot\vec{r}-\omega t)}$ where $\vec{k}_1\cdot\vec{r} = (+y\cos\theta_0 + z\sin\theta_0)k_0$

But $\vec{E}_{0y}+\vec{E}_{1y}=0$ also at $y=b$: therefore the distance between the planes along $\vec{k}_0$ or $\vec{k}_1$ is $\frac{1}{2}$ integer wavelengths:

$$k_0\left(b\cos\theta_0\right)=m\pi\,,\quad m=1,2,3,\ldots \text{ mode number}$$

Total field is $\vec{E}=\vec{E}_0+\vec{E}_1=\vec{e}_x E_{00}e^{-i\omega t}\left(e^{i\vec{k}_0\cdot\vec{r}}-e^{i\vec{k}_1\cdot\vec{r}}\right)$

$$=-2i\,\vec{e}_x E_{00}e^{-i\omega t}\sin\left(k_0 y\cos\theta_0\right)e^{ik_0 z\sin\theta_0}$$

$$\vec{B}=\vec{B}_0+\vec{B}_1=E_{00}e^{-i\omega t}\left[e^{i\vec{k}_0\cdot\vec{r}}\left(\vec{e}_y\sin\theta_0+\vec{e}_z\cos\theta_0\right)+\left(-\vec{e}_y\sin\theta_0+\vec{e}_z\cos\theta_0\right)e^{i\vec{k}_1\cdot\vec{r}}\right]$$

so B_z is generally nonzero $\rightarrow$ this is <u>not</u> a TM wave.

In the transverse direction, field has $\sin\left(k_0\cos\theta_0 y\right)$ behavior $\Rightarrow$ "standing" wave, with effective $k=k_0\cos\theta_0=\frac{m\pi}{b}\equiv k_c$

This is called the cutoff wave number.

Longitudinally, there is a traveling wave with wave number $k_g=k_0\sin\theta_0$:

and thus $k_0^2=k_c^2+k_g^2$.

Phase and group velocity: the phase velocity, $\frac{\omega}{k_g}$, in the longitudinal direction, is $u_\phi=\frac{\omega}{k_0\sin\theta_0}=\frac{c}{\sin\theta_0}\rightarrow$ clearly $>c$.

But group velocity is $u_g=c\sin\theta_0$ (velocity of wave along diagonal, in z-component).