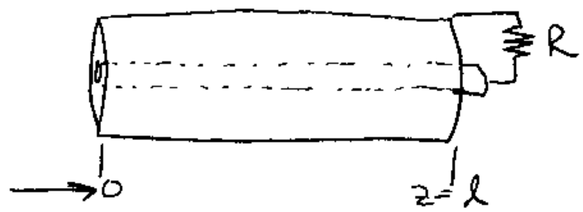
PJ320 LECTURE 21

Transmission lines: What happens at the end?

Place a <u>terminating</u> <u>resistor</u> R:



Incoming

$V_+ = V_{+0} e^{i(kz-\omega t)}$ where $\frac{V}{i} = Z_0 = \sqrt{\frac{L_\ell}{C_\ell}}$ for propagating waves.

Resistor enforces $v(\ell,t) = i(\ell,t)R$. Can satisfy this by introducing a reflected wave $V_- = V_{-0} e^{i(-kz-\omega t)}$ where $V_-$ may have a phase shift, and $\frac{V_{+0}}{I_{+0}} = \frac{-V_{-0}}{I_{-0}} = Z_0$.

But $\frac{v(\ell,t)}{i(\ell,t)} = R = \frac{V_+ e^{ik\ell} \mp V_- e^{-ik\ell}}{I_+ e^{ik\ell} + I_- e^{-ik\ell}}$ (eliminating $e^{-i\omega t}$.)

so $R = \frac{Z_0 \left(I_{+0} e^{ik\ell} + I_{-0} e^{-ik\ell}\right)}{I_{+0} e^{ik\ell} + I_{-0} e^{-ik\ell}}$

Define an "input" impedance, $Z_g = \frac{v(0,t)}{i(0,t)} = \frac{V_{+0} + V_{-0}}{I_{+0} + I_{-0}} = \frac{Z_0 (I_{+0} - I_{-0})}{(I_{+0} + I_{-0})}$

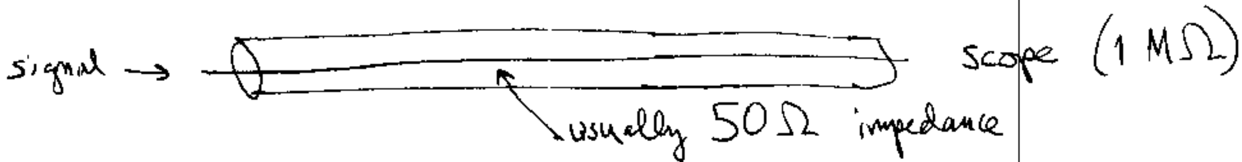
Solving for $\frac{I_{-0}}{I_{+0}}$: $Z_0 \left(I_{+0} e^{ik\ell} - I_{-0} e^{-ik\ell}\right) = R\left(I_{+0} e^{ik\ell} + I_{-0} e^{-ik\ell}\right)$

$Z_g (I_{+0} + I_{-0}) = Z_0 (I_{+0} - I_{-0})$

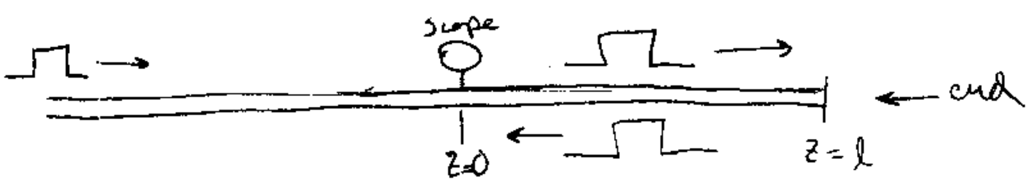
$$\Longrightarrow \frac{I_{-0}}{I_{+0}} = \frac{Z_0 e^{+ik\ell} - R e^{+ik\ell}}{Z_0 e^{-ik\ell} + R e^{-ik\ell}} = \frac{Z_0 - Z_g}{Z_0 + Z_g}$$

From which one can find the amplitude ratio: $\frac{V_{-0} e^{-ik\ell}}{V_{+0} e^{+ik\ell}} = \frac{R - Z_0}{R + Z_0} \Rightarrow \begin{matrix} -1 \text{ for } R \to 0 \\ +1 \text{ for } R \to \infty \end{matrix}$

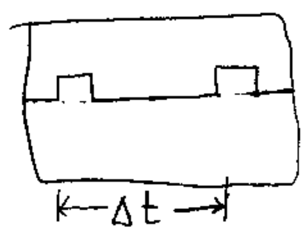
What does this mean in practice? Many applications, but one you will likely encounter is looking at "fast" (ns → μs) signals on an oscilloscope:




BNC connector on scope is carefully (expensively) matched to be 50Ω, but scope input is 1MΩ (effectively ∞). If you connect directly, there will be a negative reflected wave! Can see this by using a T connector:




open end:
Scope trace will be:




where $\Delta t = \dfrac{2l}{u}$

If you short the end:

if you put 50Ω resistor: