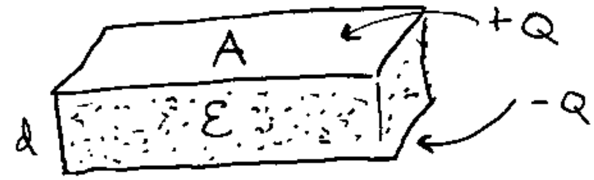
... continuing review of last semester

Parallel plate capacitor:

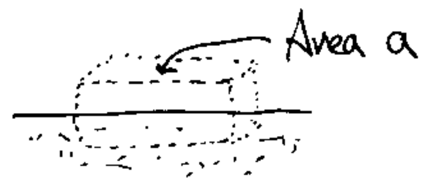


Assume no fringe fields

What is C?

Free Surface charges $\rho_{sf} = \frac{\pm Q}{A}$

Use Gaussian "pillbox" to find $\vec{D}$:



$$\int d\vec{A}\cdot\vec{D} = 4\pi Q_{f\,enc} = 4\pi \rho_{sf}\, a$$

$$\vec{D}_{up} = 0,\quad a\cdot\vec{D}_{dn} = 4\pi\rho_{sf}\, a \;\Rightarrow\; \vec{D}_{dn} = -4\pi\frac{Q}{A}\vec{e}_z$$

But $\vec{D} = \varepsilon\vec{E}$ for linear material, so $\vec{E} = -4\pi\frac{Q}{\varepsilon A}\vec{e}_z$

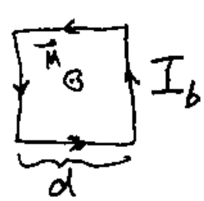
$$C = \frac{Q}{V} \qquad V = \int_d^0 d\vec{l}\cdot\vec{E} = 4\pi\frac{Qd}{\varepsilon A}$$

so $\frac{Q}{V} = \boxed{C = \frac{\varepsilon A}{4\pi d}}$ (Note the unit of capacitance is cm.)

Moving on to magnetic materials:

Magnetization $\vec{M} = \frac{\text{dipole moment}}{\text{volume}}$ as before.

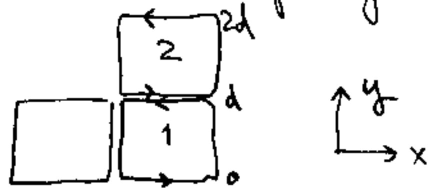
Interpreting $\vec{M}$ as bound currents, take a cube of material aligned with $\vec{M}$:



Magnetic moment of cube is $\frac{I_b}{c}d^2$ which must equal $MV = Md^3$.

So $I_b = Mdc$.

If neighbor cubes in material have same $\vec{M}$, then bound currents cancel. If not, then (ignoring higher derivatives) net current on a wall is:



$$I_{1up} = -M_z(0,0,0)dc \qquad I_{2dn} = +M_z(0,d,0)\,dc = \left[M_z(0,0,0) + \frac{\partial M_z}{\partial y} d\right]dc$$

so $$I_{net} = I_{1up} + I_{2dn} = \frac{\partial M_z}{\partial y} d^2 c$$ in x direction.

Now, use superposition to interpret this as x component of bound current density. Take d to be unit length:

$$J_{xb} = c\frac{\partial M_z}{\partial y} \therefore \quad \boxed{\vec{J}_b = c \,\mathrm{curl}\,\vec{M}}$$

Taking total $\vec{J} = \vec{J}_b + \vec{J}_f$, and knowing $\mathrm{curl}\,\vec{B} = \frac{4\pi \vec{J}}{c}$, can write

$$\mathrm{curl}\ \vec{B} = \frac{4\pi}{c}\left(\vec{J}_f + \vec{J}_b\right) \qquad \mathrm{curl}\,\vec{M} = \frac{\vec{J}_b}{c}$$

$$\text{so } \mathrm{curl}\left(\vec{B} - 4\pi\vec{M}\right) = \frac{4\pi}{c} J_f \quad \Longrightarrow \vec{H} \equiv \vec{B} - 4\pi\vec{M}.$$

Linear materials: $\vec{M} = \chi_m \vec{H}$

$$\text{so } \vec{B} = \vec{H} + 4\pi\vec{M} = \vec{H}(1 + 4\pi\chi_m) \equiv \mu\vec{H}$$

μ = permeability.

- $\mu < 1$ diamagnetism (χ_m negative)
- $\mu > 1$ paramagnetism (χ_m positive)
- $\mu \gg 1$ ferromagnetism.

Boundary conditions on $\vec{E}, \vec{D}, \vec{H}, \vec{B}$:

Remember that divergence affects <u>normal</u> components at boundaries,
curl " <u>transverse</u> " " ".

(see divergence theorem, Stokes's theorem!)

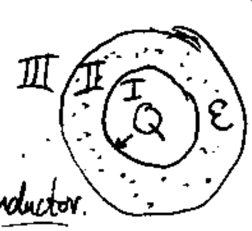
$\vec{E}$: $\left.\begin{array}{l} \operatorname{div}\vec{E} = 4\pi(\rho_b+\rho_f) \\ \operatorname{curl}\vec{E} = 0 \end{array}\right\}$ $E_\perp$ continuous only if no bound <u>or</u> free charge
$E_\parallel$ always continuous

$\vec{D}$: $\left.\begin{array}{l} \operatorname{div}\vec{D} = 4\pi\rho_f \\ \operatorname{curl}\vec{D} = 4\pi \operatorname{curl}\vec{P} \end{array}\right\}$ $D_\perp$ is continuous if no <u>free</u> charge
$D_\parallel$ continuous if $P_\parallel$ continuous.

$\vec{H}$: $\left.\begin{array}{l} \operatorname{div}\vec{H} = -4\pi \operatorname{div}\vec{M} \\ \operatorname{curl}\vec{H} = \frac{4\pi}{c}\vec{J}_f \end{array}\right\}$ $H_\perp$ is continuous if $\vec{M}_\perp$ continuous
$H_\parallel$ continuous if no <u>free</u> current.

$\vec{B}$: $\left.\begin{array}{l} \operatorname{div}\vec{B} = 0 \\ \operatorname{curl}\vec{B} = \frac{4\pi}{c}(\vec{J}_f+\vec{J}_b) \end{array}\right\}$ $B_\perp$ is <u>always</u> continuous
$B_\parallel$ continuous if no free <u>or</u> bound current

Examples: Charged conducting sphere with dielectric coating:



Q is (free) charge on <u>conductor</u>.

Outer surface: No free charge $\Rightarrow D_r$ continuous
Bound charge $\Rightarrow E_r$ discontinuous

Inner surface: Bound + free charge: D, E discontinuous.

Region I: $D, E = 0$ by symmetry

Region II: Gaussian surface, radius r: $\oint d\vec{A}\cdot\vec{E} = 4\pi(Q+Q_b)$, $\oint d\vec{A}\cdot\vec{D} = 4\pi Q$

So, since fields are radial by symmetry,

$$\vec{E} = \frac{Q+Q_b}{r^2}\vec{e}_r, \quad \vec{D} = \frac{Q}{r^2}\vec{e}_r \quad \text{but since } \vec{D} = \varepsilon\vec{E}, \text{ know}$$

$$\frac{Q+Q_b}{r^2} = \frac{Q}{\varepsilon r^2} \implies Q_b = Q\left(\frac{1}{\varepsilon} - 1\right).$$

Region III: Since dielectric has no net charge, take total $Q_{encl} = Q$.

$$\Rightarrow \vec{E} = \frac{Q}{r}\vec{e}_r, \quad \vec{D} = \frac{Q}{r}\vec{e}_r.$$

In cartoon form: