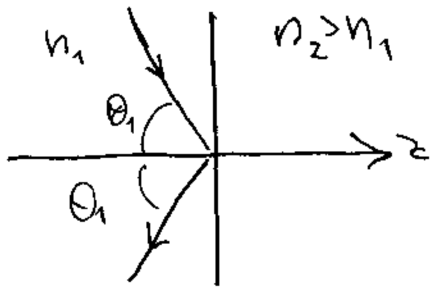
Total internal reflection: the evanescent wave



$\theta_1 > \theta_c$ where $\sin\theta_c = \frac{n_2}{n_1}$

How to treat the field in the $z>0$ region: $\vec{E}_2 = \vec{E}_2^{\,0} e^{i(\vec{k}_2 \cdot \vec{r} - \omega t)}$

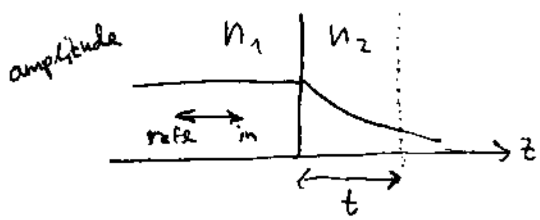
First, treat $\vec{k}_2$ vector:

$$\vec{k}_2 \cdot \vec{r} = \left(iQz - Wx \right) k_2 \quad \text{where "}\sin\theta_2\text{"} = \frac{\sin\theta_1}{\sin\theta_c} = W > 1$$

(- sign due to def'n of θ_2)

$$\text{"}\cos\theta_2\text{"} = \sqrt{1-\sin^2\theta_2} = iQ$$

Now, $\vec{E}_2 = \vec{E}_2^{\,0} e^{-i\omega t} e^{-iWx} e^{-Qz}$, so field decays in z, but "propagates" in x, parallel to the surface: this is called an evanescent wave:



What if there is another region of n_1 on the other side, at distance $z = +t$? Boundary conditions will require the field to penetrate into the third region if e^{-Qt} is not too small. But here, the field looks like a z-reversed version of the first interface — so an (attenuated) propagating wave will emerge at $z = t$, going into the right-hand n_1 region.

Reflection/refraction off conductors:

Recall definition of complex dielectric constant $\hat{\varepsilon} = \varepsilon + \frac{4\pi i \sigma}{\omega}$ where σ = conductivity, and then complex refractive index $\hat{n} \equiv \sqrt{\hat{\varepsilon}\mu}$

For normal incidence, extend the discussion of reflected/transmitted field:

$$E^0_{ref} = \frac{\hat{n}_2 - n_1}{\hat{n}_2 + n_1} E^0_{in}$$

$$E^0_{tr} = \frac{2n_1}{\hat{n}_2 + n_1} E^0_{in}$$

$\Longrightarrow$ Note: — E^0_{ref} and E^0_{tr} have phase shifts that are <u>not</u> π and 0 now.

Assuming electrical properties of medium 2 are dominated by conduction: $\hat{n}_2 \approx \sqrt{\frac{4\pi i \sigma_2}{\omega}} = \frac{c}{\omega\delta}(1+i)$ where $\delta = \frac{c}{\sqrt{2\pi\sigma\omega}}$

$|\hat{n}_2|$ can now become <u>very</u> large if conductor is good. In this limit, $R = \left|\frac{E^0_{ref}}{E^0_{in}}\right|^2 \rightarrow \left|\frac{\hat{n}_2 - 1}{\hat{n}_2 + 1}\right|^2 \longrightarrow 1$.

In less extreme case, can find reflection coefficient: H&M define "reduced wavelength" $\lambda\!\!\!^{-} \equiv \frac{c}{\omega}$. In vacuum, $\lambda\!\!\!^{-} = \frac{1}{k}$.

Now, $$E^0_{ref} = \frac{(1+i)\frac{\lambda\!\!\!^{-}}{\delta} - n_1}{(1+i)\frac{\lambda\!\!\!^{-}}{\delta} + n_1} E^0_{in}$$

$$\Longrightarrow R = \frac{|E_{ref}|^2}{|E_{in}|^2} = \frac{\left|1+i-n_1\frac{\delta}{\lambda\!\!\!^{-}}\right|^2}{\left|1+i+n_1\frac{\delta}{\lambda\!\!\!^{-}}\right|^2} = \frac{1+\left(1-\frac{\delta}{\lambda\!\!\!^{-}}n_1\right)^2}{1+\left(1+\frac{\delta}{\lambda\!\!\!^{-}}n_1\right)^2}$$

... which can be expanded in $\frac{\delta}{\lambda}$, which is small for good conductors:

$$R \approx 1 - 2\frac{\delta}{\lambda} n_1$$

If $n_1 = 1$, then $T = 2\frac{\delta}{\lambda} = 2\delta k_0$.