Fresnel Equations: for dielectric (not magnetic) materials:

Fields for transmitted and reflected polarized waves

Polarization normal to plane of incidence (parallel to material boundary):

$$\frac{E^0_{tr}}{E^0_{in}} = \frac{2\cos\theta_1}{\cos\theta_1 + \frac{n_2}{n_1}\cos\theta_2} = \frac{2\cos\theta_1 \sin\theta_2}{\sin(\theta_1+\theta_2)}$$

$$\frac{E^0_{ref}}{E^0_{in}} = \frac{\cos\theta_1 - \frac{n_2}{n_1}\cos\theta_2}{\cos\theta_1 + \frac{n_2}{n_1}\cos\theta_2} = \frac{\sin(\theta_2-\theta_1)}{\sin(\theta_2+\theta_1)}$$

Polarization parallel to plane of incidence:

$$\frac{E^0_{tr}}{E^0_{in}} = \frac{2\cos\theta_1}{\cos\theta_2 + \frac{n_2}{n_1}\cos\theta_1} = \frac{2\cos\theta_1\sin\theta_2}{\sin(\theta_1+\theta_2)\cos(\theta_1-\theta_2)}$$

$$\frac{E^0_{ref}}{E^0_{in}} = \frac{\cos\theta_1 - \frac{n_1}{n_2}\cos\theta_2}{\cos\theta_1 + \frac{n_1}{n_2}\cos\theta_2} = \frac{\tan(\theta_1-\theta_2)}{\tan(\theta_1+\theta_2)}$$

Look at consequences:

— First, parallel and normal polarizations clearly have different reflection/transmission coefficients. But if $\vec{E}$ is parallel to plane of incidence, $\vec{H}$ is normal and vice-versa. If the boundary conditions on $\vec{E}$ and $\vec{H}$ are identical, how can it matter if $\vec{E}$ or $\vec{B}$ is polarized in a given direction?

$\Longrightarrow$ we assumed dielectric, non-magnetic media in the calculation. Can generalize using $n \to \frac{1}{\eta}$.

Normal polarization: no critical angles except $\theta_1 = 0, \frac{\pi}{2}, \theta_2$.

Transverse polarization: $E_{ref} = 0$ if either $\theta_1 = \theta_2$ or

$\theta_1 + \theta_2 = \frac{\pi}{2}$ where denominator diverges.

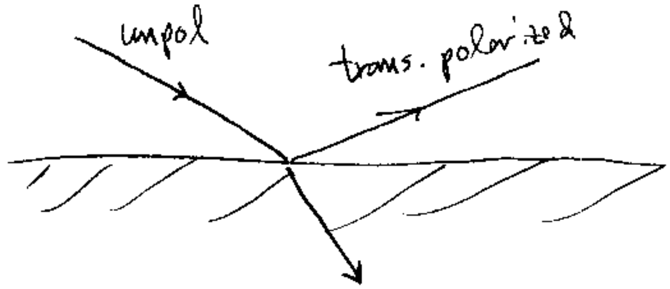
This critical angle is called Brewster's angle, $\theta_1 = \theta_B$.

If $\theta_2 = \frac{\pi}{2} - \theta_1$ and $n_1 \sin\theta_1 = n_2 \sin\theta_2 = n_2 \cos\theta_1$

then
$$\frac{n_2}{n_1} = \frac{\sin\theta_B}{\cos\theta_B} = \tan\theta_B$$

At this angle, transverse-polarized waves are not reflected at all. Unpolarized light incident on surface at θ_B will have only normal-polarized light reflected.

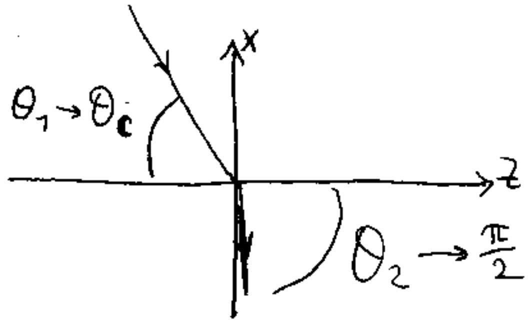
In practice, reflection coefficients for normal-polarization are much higher than for transverse at most angles away from 0 or $\frac{\pi}{2}$.



Total internal reflection: consider wave going from n_1 to n_2 where $n_1 > n_2$ (glass or water $\rightarrow$ air):

Snell's Law: $n_1 \sin\theta_1 = n_2 \sin\theta_2$

At $\theta_1 = \sin^{-1}\left(\frac{n_2}{n_1}\right)$, θ_2 becomes $\pi/2$:



If $\theta_1 > \theta_c$, $\cos\theta_2$ becomes pure imaginary:

$$\cos\theta_2 = \sqrt{1-\sin^2\theta_2}$$
$$= \sqrt{1-\left(\frac{n_1}{n_2}\sin\theta_1\right)^2}$$
$$= \sqrt{1-\left(\frac{\sin\theta_1}{\sin\theta_c}\right)^2}$$

What is the wave's behavior in the n_2 medium?

$$\vec{E}_2 = \vec{E}_2^{\,0}\exp[i(\vec{k}_2\cdot\vec{r}-\omega t)] \qquad \text{(can't really call it } \vec{E}_{tr}\text{)}$$

where $\vec{k}_2\cdot\vec{r} = \left(-k_2\sin\theta_2\right)x + \left(k_2\cos\theta_2\right)z$

Since $\cos\theta_2$ is imaginary, this gives an $e^{-\lambda z}$ factor, so the field decays in z but propagates in x:

$$\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1 = \frac{\sin\theta_1}{\sin\theta_c} \equiv W \quad \text{in H\&M notation}$$

$$\cos\theta_2 = \sqrt{1-\left(\frac{\sin\theta_1}{\sin\theta_c}\right)^2} \equiv iQ \qquad \text{so}$$

$$\vec{E}_2 = \vec{E}_2^{\,0}\exp(-k_2 Q z)\exp\left[i\left(-k_2 W x-\omega t\right)\right]$$

and the phase velocity in x is

$$\frac{\omega}{k_2 W} = \frac{\omega}{k_2}\,\frac{n_2}{n_1\sin\theta_1} = \frac{c}{n_1\sin\theta_1}$$