P3320 LECTURE 16

Exam details: 7 - 8^{30} pm, Thurs, G2B47.

Ground rules: 1 crib sheet, no electronics, no textbook.

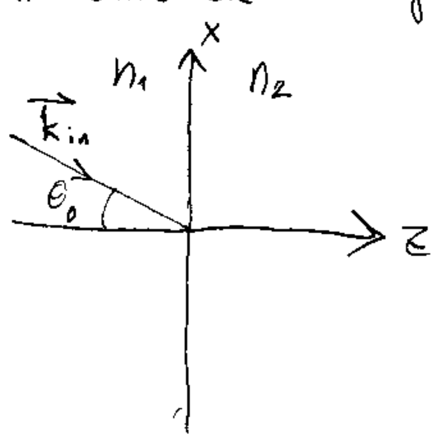
Back to wave transmission/reflection at boundaries. Did normal incidence on Friday; found:

- Transmitted wave has no phase shift
- Reflected wave has phase shift of π in:

  $\vec{E}$ if going into denser medium $(n_2 > n_1)$

  $\vec{B}$ if going into less dense medium $(n_1 > n_2)$
- Power transmitted is $\dfrac{P_{trans}}{P_{in}} = \dfrac{4 n_1 n_2}{(n_1+n_2)^2}$.

Now, consider incidence at an angle: boundary is the x-y plane



Define $\vec{k_{in}}$ in the x-z plane. Symmetry requires reflected, transmitted waves also propagate in x-z plane. Call the reflected, transmitted waves $\vec{k}_{ref}$, $\vec{k}_{tr}$: Now:

$$\vec{E}_{in} = \vec{E}^{\circ}_{in} \exp[i(\vec{k}_{in}\cdot\vec{r} - \omega t)]$$

$$\vec{H}_{in} = \vec{H}^{\circ}_{in} \exp[i(\vec{k}_{in}\cdot\vec{r} - \omega t)] \quad \text{where} \quad \frac{|\vec{H}^{\circ}_{in}|}{|\vec{E}^{\circ}_{in}|} = n_1 \quad \text{and} \quad \frac{\vec{E}^{\circ}_{in} \times \vec{H}^{\circ}_{in}}{|\vec{E}^{\circ}_{in}|\,|\vec{H}^{\circ}_{in}|} = \vec{e}_{k_{in}}$$

$$\Longrightarrow \vec{H}_{in} = n_1 \vec{e}_{k_{in}} \times \vec{E}_{in}.$$

Similarly, $\vec{E}_{ref} = \vec{E}^{\circ}_{ref} \exp\left[i\left(\vec{k}_{ref}\cdot\vec{r} - \omega t\right)\right]$

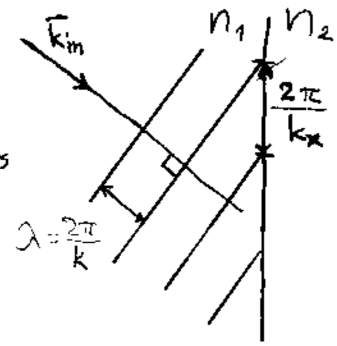
$$\vec{E}_{tr} = \vec{E}^{\circ}_{tr} \exp\left[i\left(\vec{k}_{tr}\cdot\vec{r} - \omega t\right)\right]$$

$$\vec{H}_{ref} = n_1 \vec{e}_{kref} \times \vec{E}_{ref}$$

$$\vec{H}_{tr} = n_2 \vec{e}_{ktr} \times \vec{E}_{tr}$$

Note — why use $\vec{H}$ instead of $\vec{B}$? Boundary conditions on $\vec{H}$ are easy to use, since no $\vec{J}_{free} \Rightarrow H_{\parallel}$ continuous.

Now, consider plane wavefronts: planes of constant phase

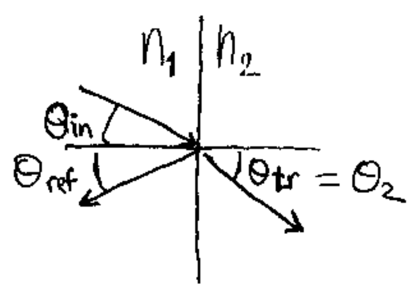


On boundary, these wavefronts must have same spatial periodicity for $\vec{k}_{in}, \vec{k}_{ref}, \vec{k}_{tr}$,

since boundary conditions require that $E_{\parallel 1} = E_{\parallel 2}$ at all points on the plane.

$\Rightarrow k_{in\,x} = k_{ref\,x} = k_{tr\,x}$.

Define angles of waves:



$k_{inx} = k_{in} \sin\theta_{in}$

$k_{refx} = k_{ref} \sin\theta_{ref}$

$k_{trx} = k_{tr} \sin\theta_{tr}$

Since ω can't change, $|\vec{k}_{in}| = |\vec{k}_{ref}| = k_1$ and $|\vec{k}_{tr}| = k_2$

Thus $\theta_{ref} = \theta_{in} = \theta_1$

and $k_1 \sin\theta_1 = k_2 \sin\theta_2 \Rightarrow$ Since $k = \frac{n\omega}{c}$ and ω fixed,

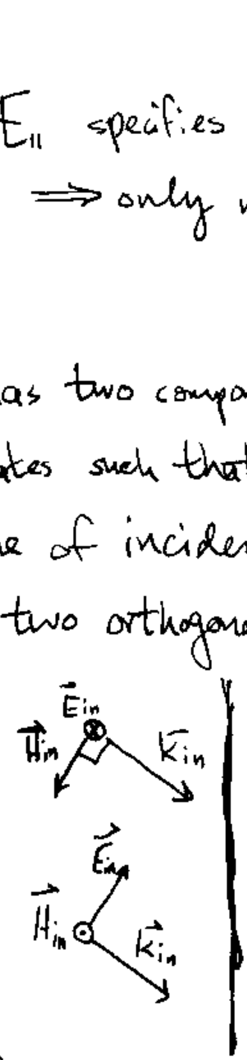
$n_1 \sin\theta_1 = n_2 \sin\theta_2$ Snell's Law.

Now, look at constraints on the polarization and power in the transmitted/reflected waves. These come from boundary conditions:

$$D_{\perp in} + D_{\perp ref} = D_{\perp tr}$$
$$E_{\parallel in} + E_{\parallel ref} = E_{\parallel tr}$$
$$B_{\perp in} + B_{\perp ref} = B_{\perp tr}$$
$$H_{\parallel in} + H_{\parallel ref} = H_{\parallel tr}$$

Note that specifying $|k|, \theta,$ and $E_\parallel$ specifies $E_\perp = \frac{1}{\varepsilon} D_\perp$, and similarly for $B_\perp$ and $H_\parallel$ $\Rightarrow$ only need to work with $E_\parallel$ and $H_\parallel$.

Now, consider polarizations: $E_\parallel$ has two components, E_x and E_y. (Recall we have selected the coordinates such that the $\vec{k}$ vectors are all in the x-z plane – the "plane of incidence"). Can describe the polarization as superposition of two orthogonal polarization states:



- $E_\parallel = E_y$, $E_\perp = 0$:

(Polarization normal to plane of incidence)

- $E_\parallel = E_x$, $E_\perp = E_x \tan\theta$

(Polarization parallel to plane of incidence)

Normal case: $E_{\parallel in}(z=0) + E_{\parallel ref}(z=0) = E_{\parallel tr}(z=0)$.
Since $E_\perp = 0$, and this must be true at all points on plane, know that $E^0_{in} + E^0_{ref} = E^0_{tr}$, and the three waves have the same polarization. More detail in textbook.