PHYS3320 LECTURE 11

Problem when multiplying complex fields: if $\vec{E} = \mathrm{Re}\left(\vec{E_0} e^{i\omega t}\right)$ then $E^2 \neq \mathrm{Re}\left[(\vec{E}_0 e^{i\omega t})^2\right]$. <u>But</u> time averages obey a much simpler relation: if representing one field as $F = \mathrm{Re}\left(F_0 e^{i\omega t}\right)$ and $G = \mathrm{Re}\left(G_0 e^{i(\omega t + \delta)}\right)$ then time average

$$\langle FG \rangle = F_0 G_0 \left\langle (\cos\omega t)(\cos\omega t \cos\delta - \sin\omega t \sin\delta) \right\rangle$$

$$= F_0 G_0 \left(\frac{1}{2}\cos\delta\right)$$

$$\ldots \text{which is } \frac{1}{2}\mathrm{Re}\left[\left(F_0 e^{i\omega t}\right)\left(G_0 e^{i(\omega t+\delta)}\right)^*\right]$$

So when doing averages, take real part implicitly: $\langle FG \rangle = \frac{1}{2} F_0 G_0^* = \frac{1}{2} F_0^* G_0$

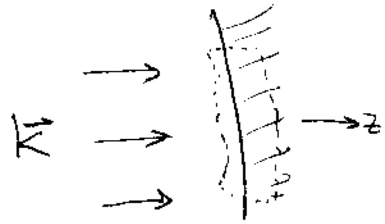
Therefore, average energy in Poynting vector is:

$$\langle \vec{S} \rangle = \frac{1}{2}\, \frac{c}{4\pi} E_0 \times H_0^* = \frac{c}{8\pi} \frac{E_0^2}{\eta} \vec{e}_k$$

And energy density is $\frac{1}{16\pi}\left(\varepsilon E_0^2 + \mu H_0^2\right)$, since it must be $\frac{1}{8\pi}\left(\varepsilon E^2 + \mu H^2\right)$ instantaneously, but $\langle E^2 \rangle = \frac{E_0^2}{2}$, etc.

Radiation pressure: Radiation absorbed by a surface exerts a pressure on it. This can be calculated using a few techniques, including momentum density via Poynting vector, and net force using the Maxwell stress tensor. (Note: Assume waves in vacuum.)

Nicest (conceptually) is using Maxwell stress tensor, with an integration surface encompassing the absorbent surface:



Left side: field is the standard radiation

Right side: field absorbed $\Rightarrow E = B = 0$.

Net force on the enclosed volume is pressure $\cdot$ area, also equal to $\int \mathbb{T} \cdot d\vec{A}$ on left face of surf.

$$\langle \vec{F} \rangle = \left\langle -\int \mathbb{T} \cdot d\vec{A} \right\rangle \qquad \text{where } \vec{E} = E_0 \vec{e}_x e^{i(kz-\omega t)},\ \vec{B} = B_0 e^{i(kz-\omega t)} \vec{e}_y$$

and $d\vec{A} = \vec{e}_z\, dA$

so

$$\langle \mathbb{T} \rangle \Leftrightarrow \frac{1}{4\pi} \begin{pmatrix} +\frac{1}{4}(E_0^2+B_0^2) & & 0 \\ & +\frac{1}{4}(E_0^2+B_0^2) & \\ 0 & & -\frac{1}{4}(E_0^2+B_0^2) \end{pmatrix}$$

so $\dfrac{\langle \vec{F} \rangle}{A} = -\langle T_{zz} \rangle \vec{e}_z = +\dfrac{1}{16\pi}\left(E_0^2 + B_0^2\right) = \dfrac{1}{8\pi} E_0^2$ because we're in vacuum.

$\rightarrow$ Can also be shown using Poynting vector, as wave's momentum density is $\frac{1}{c^2}\vec{S}$, and it's "delivered" to its target at speed c.

(Note units of pressure $= \dfrac{\text{dynes}}{\text{cm}^2}$.)

What is field strength, radiation pressure from a light bulb?

Take 100W bulb, say 10% efficient, at $r = 100$ cm.

$10\ W = 10^8 \frac{\text{erg}}{\text{sec}}$, so radiative energy flux at 100 cm is

$$\frac{10^8\ \text{erg/sec}}{4\pi r^2} = \frac{c}{4\pi} \left| \vec{E} \times \vec{B} \right| \quad \text{(time average, that is)}$$

so $\frac{c}{8\pi} E_0^2 = \frac{10^8 \text{ erg/s}}{4\pi (10^4 \text{ cm}^2)}$

$$\Rightarrow E_0^2 = \frac{2 \cdot 10^4 (\text{erg/s})}{3 \cdot 10^{10} \text{ cm}^3/\text{s}} = \frac{2}{3} 10^{-6} \left(\frac{\text{stV}}{\text{cm}}\right)^2$$

These units are equivalent

$$\Longrightarrow B_0^2 = \frac{2}{3} \times 10^{-6} \; G^2$$

... so we're talking milligauss fields, vs. $\sim 1G$ for earth's $\vec{B}$. In SI units, this is $\sim 300 \frac{V}{m}$, so it's not <u>that</u> tiny. But since the field direction is all over the place, nothing gets sparky.

Radiation pressure $= \frac{1}{8\pi} E_0^2 \cong \frac{1}{40} \cdot 10^{-6} \frac{\text{dyne}}{\text{cm}^2}$

$$= 0.25 \cdot 10^{-13} \text{ atmosphere!}$$