P3320 LECTURE 1

Last semester: physics of static electromagnetic fields

This semester: fields that change with time.

Course schedules, policy same as last semester. Exams: 10/2, 11/6 ?

First — some notation changes from last semester:

* Gaussian units! See lecture 42 from spring.

Heald & Marion use slightly different notation:

- $\vec{r}-\vec{r}'$ explicitly instead of $\vec{\mathscr{r}}$
- $\vec{e}_x$, $\vec{e}_y$, etc. for unit vectors instead of $\hat{x}$, $\hat{y}$, ...

$\Rightarrow$ klutzy, but at least $\vec{e}_{r-r'}$ is better than $\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|}$.

- $\vec{u}$ instead of $\vec{v}$ for velocity

(I may continue to use $\mathscr{r}$, $\hat{x}$, etc. in lecture when it's more convenient!)

- surface charge ρ_s, line charge ρ_ℓ.
- r not s for cylindrical radius.

Recapping key points from spring semester:

Static Fields: $\text{div}\,\vec{E} = 4\pi\rho$, $\text{div}\,\vec{B} = 0$

$\text{curl}\,\vec{E} = 0$, $\text{curl}\,\vec{B} = \frac{4\pi}{c}\vec{J}$

Force laws: $\vec{F} = q\vec{E} = \frac{q_1 q_2}{\mathscr{r}^2}\hat{\mathscr{r}}$ Coulomb's Law

$\vec{F}$ in dynes, r in cm, q in esu = statcoulombs $\Rightarrow \vec{E} = \frac{q}{r^2}\hat{r}$ for q at origin.

Magnetic: $\vec{F} = q_t \frac{\vec{u}}{c} \times \vec{B}$ Lorentz force law

$$\vec{B} = \frac{1}{c} \int d^3r' \, \frac{\vec{J}(\vec{r}') \times \hat{\mathscr{r}}}{\mathscr{r}^2}$$

Biot-Savart integral:
B in gauss, $\vec{J}$ in $\frac{\text{statamp}}{\text{cm}^2} = \frac{\text{esu}}{\text{s} \cdot \text{cm}^2}$

→ Notation warning! H&M use $\vec{r}$ as a shorthand for $\vec{r}-\vec{r}'$ when it's "obvious from context." Usually means when writing something like:

$$\vec{G} = \int dr$$

↑ depends on position r

↑ integral over position space → should be $\vec{r}-\vec{r}'$

(If you don't think $r'-r$, you'll usually get $G(0)$.)

Ampere's Law: $\oint d\vec{l} \times \vec{B} = \frac{4\pi}{c} I_{link}$ ($I_{link} = I_{enclosed}$)

Gauss's Law: $\oint d\vec{A} \cdot \vec{E} = 4\pi Q_{encl}$.

Potential: $\vec{E} = -\text{grad}\, \Phi$ (possible because curl $\vec{E} = 0$)

$$-\text{div grad}\, \Phi \equiv -\nabla^2 \Phi = 4\pi\rho$$

$$\Phi(\vec{r}) - \Phi(\vec{r}_0) = -\int_{\vec{r}_0}^{\vec{r}} d\vec{l} \cdot \vec{E}$$

For a point charge, with $\vec{r}_0 = \infty$, $\Phi = \frac{q}{r}$

Generalizing this to multiple charges:

$$\vec{E}(\vec{r}) = \int d^3r' \, \frac{\rho(\vec{r}')}{|\vec{r}-\vec{r}'|^2} \vec{e}_{r-r'}$$

$$\Phi(\vec{r}) = \int d^3r' \, \frac{\rho(\vec{r}')}{|\vec{r}-\vec{r}'|}$$

Magnetic vector potential: $\vec{B} = \mathrm{curl}\,\vec{A}$.

Φ defined up to a constant offset

$\vec{A}$ defined up to the gradient of any scalar function!

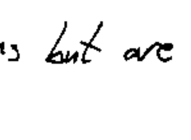
For a loop of current, in Coulomb gauge ($\mathrm{div}\,\vec{A} = 0$)

$$\vec{A}(\vec{r}) = \frac{I}{c} \oint \frac{d\vec{l}'}{|\vec{r}-\vec{r}'|}$$

For other current distributions, $\vec{A}(\vec{r}) = \dfrac{1}{c} \displaystyle\int d^3r' \frac{\vec{J}(\vec{r}')}{|\vec{r}-\vec{r}'|}$

Fields in materials:

Electric polarization $\vec{P} = \dfrac{\text{dipole moment}}{\text{volume}}$.

(Recall that for point charges 

$\vec{p} = q\vec{l}$.)

Define the displacement field $\vec{D} \equiv \vec{E} + 4\pi\vec{P}$

(Note $\vec{D}$, $\vec{P}$, $\vec{E}$ have same dimensions but are very different physically!)

Bound charges: $\rho_b = -\mathrm{div}\,\vec{P}$

since $\operatorname{div}\vec{E} = 4\pi\rho = 4\pi(\rho_f + \rho_b)$

then $\operatorname{div}(\vec{E} + 4\pi\vec{P}) = \operatorname{div}\vec{D} = 4\pi(\rho_f + \rho_b - \rho_b)$

$= 4\pi\rho_f$ ("Gauss's Law for $\vec{D}$")

Linear materials: $\vec{P} = \chi_e \vec{E}$ inside material

so $\vec{D} = \vec{E} + 4\pi\chi_e\vec{E} = (1 + 4\pi\chi_e)\vec{E}$

Define $\varepsilon \equiv 1 + 4\pi\chi_e$ "dielectric constant"

ε in MKS / SI units is different! This ε is numerically the same as $\frac{\varepsilon}{\varepsilon_0}$ in SI, but since $\vec{D} = \varepsilon\vec{E}$ here too, don't be confused! Vacuum has $\varepsilon = 1$. Tempting to say "$\varepsilon_0 = 1$" and use SI equations, but the 4π from rationalization will screw you up if you then find $\vec{P}$.