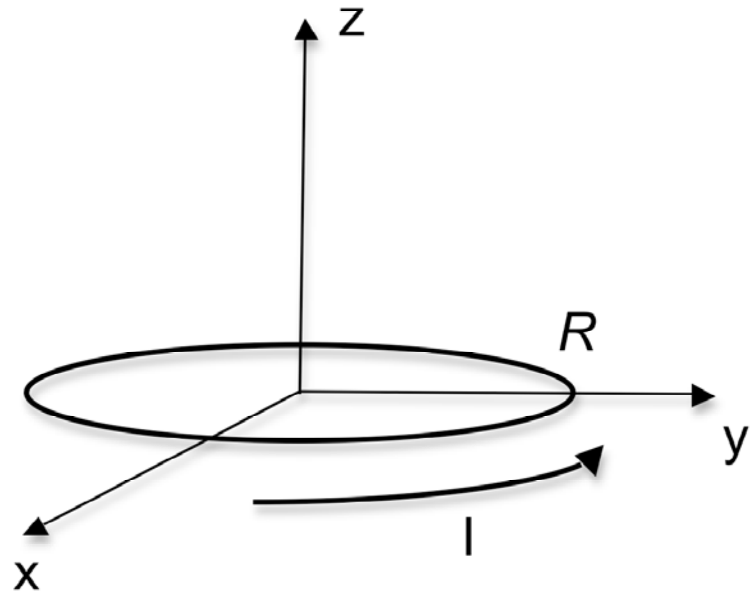
Part 1: The Biot-Savart Law and the Current Loop

Our most basic (non-infinite) current distribution is the simple circular loop of current *I* around a circular path of radius *R*, for which we *cannot* use Ampere's law to find the magnetic field **B**. We'll review how to use the Biot-Savart law by finding the field for this case, as shown in the figure.



i. The Biot-Savart law is $\mathbf{B} = \frac{\mu_0}{4\pi}\int \frac{\vec{I}\,dl' \times \hat{\mathscr{r}}}{\mathscr{r}^2}$. Describe *in words* what the vector $\mathscr{r}$ represents (not the mathematical definition in terms of other vectors).

ii. Now give a formal mathematical definition for $\mathscr{r}$ in terms of the vectors describing the position in space at which you wish to determine the field **B** and the position in space of a small "piece" of current *Idl'*. Draw these three vectors on the figure above.

iii. Now we want to move from the formal definitions to the concrete definitions we'll use to solve the problem. Find useful expressions for *Idl'* and $\mathscr{r}$ in terms of variables and component basis vectors which you can use to perform the integration for this loop, assuming you wish to find **B** at an arbitrary point (*x*,*y*,*z*).

iv. Now set up the integral using your expressions from iii. What are the limits of the integral?

v. Now evaluate your integral for **B** on the *z*-axis, that is, $(0,0,z)$. Does your answer make sense to you? Does **B** point in the expected direction? When you consider the limiting cases of $z \to 0$ and $z >> R$, does your answer behave as you expect?

vi. In particular, what should this loop look like at $z >> R$? What expression for **B** do you expect in this limit, in terms of a multipole expansion? Does your answer agree with the multipole expansion expectation?

Part 2: The Vector Potential of the Current Loop

For the same current loop, we can now quickly find the vector potential **A**,

where $\mathbf{A} = \frac{\mu_0}{4\pi}\int \frac{\vec{I}dl'}{\mathscr{r}}$.

i. Using your expressions from part 1, set up the integral to determine **A** at an arbitrary point *(x,y,z)*. Does it look simpler to evaluate than the Biot-Savart integral?

ii. Now evaluate the integral to find **A** on the *z*-axis. Does your answer for **A** make sense to you?

iii. Can you calculate **B** from this result for **A**? If so, check your result from part 1. If not, what additional information is needed to find **B** from **A**?