Part 1 – Sketching Vector Potential

One of Maxwell's equations, $\nabla \times \vec{E} = 0$, made it useful for us to define a scalar potential V, where $\vec{E} = -\nabla V$. Similarly, another one of Maxwell's equations makes it useful for us to define the vector potential, **A**.

i. Which if any of Maxwell equations makes it useful for us to define **A**? Explain.

ii. What current density **J** would create the **B**-field (uniform within a cylindrical volume) in Figure 1 below? Can you write an explicit mathematical formula for it?

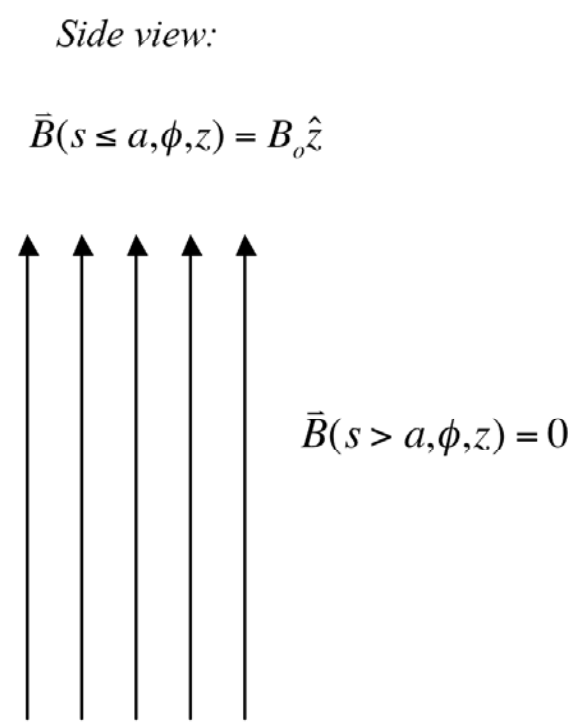


Figure 1

iii. Notice that the equations defining **A** are mathematically analogous to Maxwell's equations for **B**:

$$\nabla \bullet \vec{\mathbf{B}} = 0 \quad \Leftrightarrow \quad \nabla \bullet \vec{\mathbf{A}} = 0$$
$$\nabla \times \vec{\mathbf{B}} = \mu_0 \vec{\mathbf{J}} \quad \Leftrightarrow \quad \nabla \times \vec{\mathbf{A}} = \vec{\mathbf{B}}$$

First, sketch **B** in Figure 2 (again, note this is a “cylindrical” volume with uniform **J**). Then, using the mathematical similarities above, sketch **A** in Figure 3:

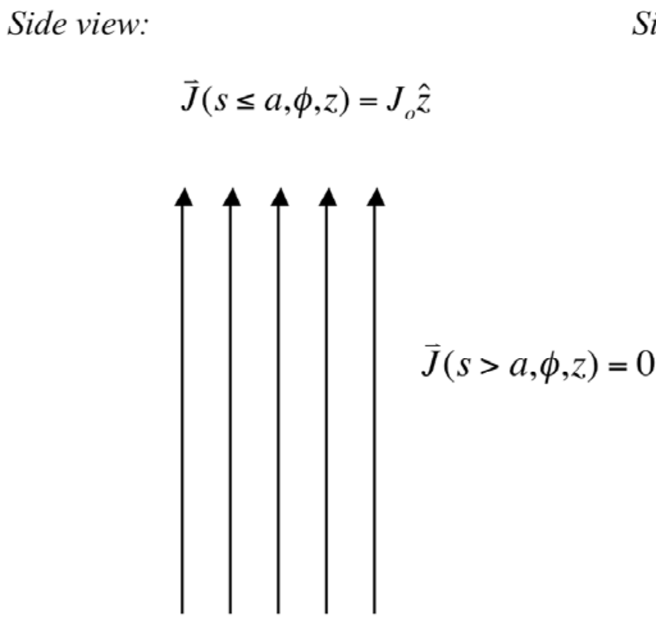


Figure 2: Given **J**, sketch the **B** field.

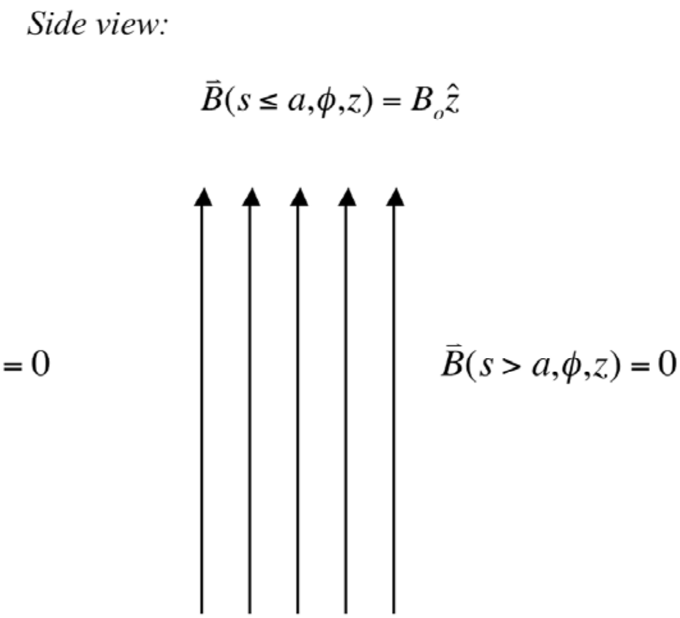


Figure 3: Given **B**, sketch the **A** field.

iv. One way to check your previous answer (conceptually) is using an Ampere’s Law analogy. Ampere’s Law tells you that the **J**-flux (or I_{encl}) is equal to $\oint \vec{B} \bullet d\vec{l}$.

What is a similar relationship between the vector potential and magnetic field?

Try using this “Ampere’s Law analogy” to (conceptually) check your sketch of **A**.

v. A toroidal wire coil looks like a doughnut wrapped with wire, as shown in Fig. 4. On the "blank" toroid, indicate the direction of **J**, then sketch the **B** and **A** fields.

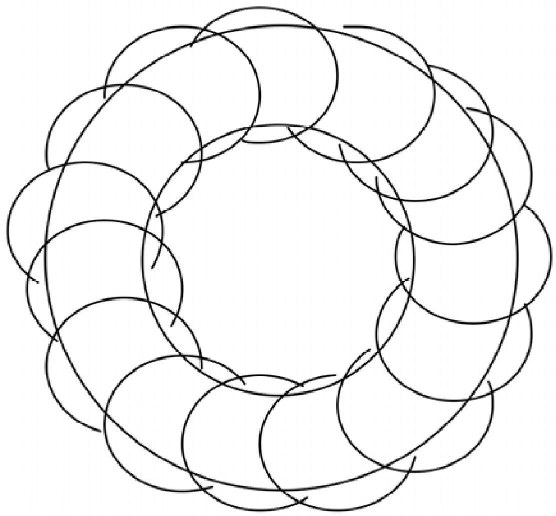
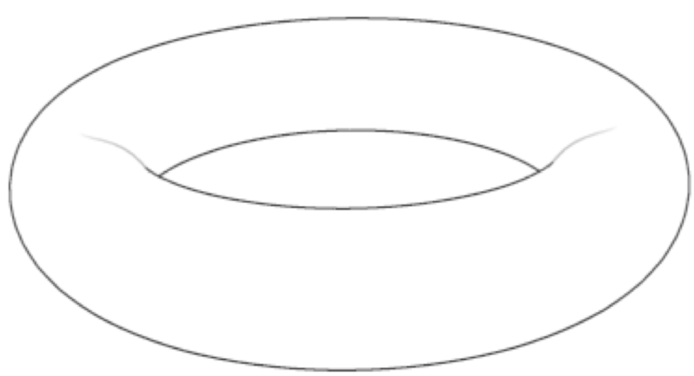


Figure 4



Part 2 – Calculating Vector Potential

On a previous homework, you calculated the magnetic field produced by a uniform surface current: $K(z=0) = K_o\hat{x}$. The answer you should have calculated is:

$$\vec{B}(z>0) = \frac{-\mu_o K_o}{2}\hat{y}$$

$$\vec{B}(z<0) = \frac{+\mu_o K_o}{2}\hat{y}$$

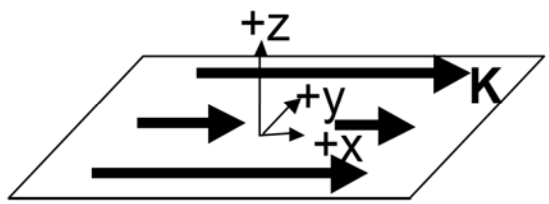
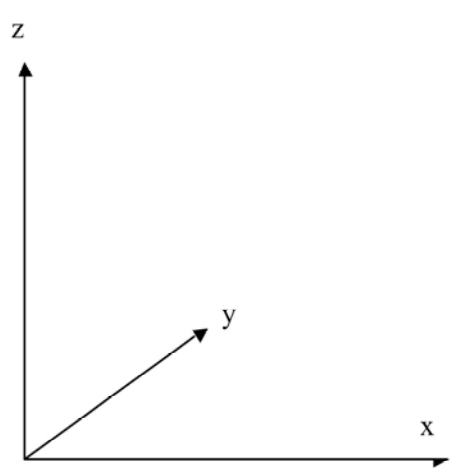


Figure 5

i. Which of the following practical devices can be modeled by the currents or fields shown in figures 2 through 5?

A: Tokamak, B: Solenoid, C: Ribbon conductor, D: Wire of finite diameter

ii. Sketch your best guess of what **A** looks like for the uniform surface current. Which components (x, y, or z) does **A** have (it might help to look at relationship between **A**, **B**, and **J** in the two examples in Part 1)? Which variables (x, y, or z) does **A** depend on?



iii. Using your assumption for which components **A** has, and which variables **A** depends on, calculate (or guess) what **A** is. Check your answer by calculating the **B** field from your vector potential **A**.

iv. Does your sketch of **A** agree with the answer you calculated (or guessed)?