

Possibly useful information, in addition to your “purple” sheet:

$$\begin{aligned} \nabla \cdot \left( \frac{\hat{\mathbf{z}}}{z^2} \right) &= 4\pi\delta^3(\mathbf{z}) & \mathbf{z} &= \mathbf{r} - \mathbf{r}' \\ \mathbf{E}(\mathbf{r}) &= \frac{1}{4\pi\epsilon_0} \int_{\mathcal{V}} \frac{\rho(\mathbf{r}') d\tau'}{z^2} \hat{\mathbf{z}} & (1+x)^\alpha &= 1 + \alpha x + \frac{\alpha(\alpha-1)}{2!} x^2 + \dots, \quad x < 1 \\ \mathbf{E}(\mathbf{r}) &= -\nabla V(\mathbf{r}) & V(b) - V(a) &= -\int_a^b \mathbf{E} \cdot d\mathbf{l} \\ \Delta V &= \frac{\Delta PE}{Q} & V(\mathbf{r}) &= \frac{1}{4\pi\epsilon_0} \int_{\mathcal{V}} \frac{\rho(\mathbf{r}') d\tau'}{z} \\ W &= \frac{1}{2} \int \rho V d\tau & W &= \frac{\epsilon_0}{2} \int E^2 d\tau \\ C &= \frac{Q}{\Delta V} & W &= \frac{1}{2} C \Delta V^2 \\ \epsilon_0 &= 8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N} \cdot \text{m}^2} & k &= \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \\ \vec{p} &= \sum_i q_i \vec{r}'_i & \vec{p} &= \int \vec{r}' \rho(\vec{r}') d\tau \\ \vec{N} &= \vec{p} \times \vec{E} & \vec{F} &= (\vec{p} \cdot \nabla) \vec{E} \\ V_{dip}(\vec{r}) &= \frac{\vec{p} \cdot \hat{\mathbf{r}}}{4\pi\epsilon_0 r^2} & \nabla \cdot \vec{D} &= \rho_f \\ \vec{D} &= \epsilon_0 \vec{E} + \vec{P} & \vec{P} &= \epsilon_0 \chi_e \vec{E} \\ \sinh kx &= \frac{e^{kx} - e^{-kx}}{2} & \cosh kx &= \frac{e^{kx} + e^{-kx}}{2} \end{aligned}$$

### Legendre Polynomials

$$\begin{aligned} P_0(x) &= 1 & P_1(x) &= x \\ P_2(x) &= \frac{1}{2}(3x^2 - 1) & P_3(x) &= \frac{1}{2}(5x^3 - 3x) \\ P_4(x) &= \frac{1}{8}(35x^4 - 30x^2 + 3) & P_5(x) &= \frac{1}{8}(63x^5 - 70x^3 + 15x) \end{aligned}$$

## General Solutions to Laplace's Equation

*Cylindrical*

$$V(r, \phi) = F_0 + F_1 \ln r + \sum_{n=1}^{\infty} \left( A_n r^n + \frac{B_n}{r^n} \right) (C_n \sin n\phi + D_n \cos n\phi)$$

*Spherical*

$$V(r, \theta) = \sum_{l=0}^{\infty} \left( A_l r^l + \frac{B_l}{r^{l+1}} \right) P_l(\cos \theta)$$

$$\frac{d^2 f}{dx^2} + k^2 f = 0 \text{ has the solution } f(x) = \sin kx, \cos kx.$$

$$\frac{d^2 f}{dx^2} - k^2 f = 0 \text{ has the solution } f(x) = e^{+kx}, e^{-kx}.$$

### Multipole Expansion

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \sum_{l=0}^{\infty} \frac{1}{r^{l+1}} \int (r')^l P_l(\cos \theta_{rr'}) \rho(\vec{r}') d\tau$$

### Integrals

$$\int_0^a \sin\left(\frac{n\pi x}{a}\right) \sin\left(\frac{m\pi x}{a}\right) dx = \begin{cases} \frac{a}{2} & \text{if } n = m \\ 0 & \text{otherwise} \end{cases}$$

$$\int_0^a \cos\left(\frac{n\pi x}{a}\right) \cos\left(\frac{m\pi x}{a}\right) dx = \begin{cases} \frac{a}{2} & \text{if } n = m \\ 0 & \text{otherwise} \end{cases}$$

$$\int_{-1}^1 P_l(x) P_m(x) dx = \begin{cases} \frac{2}{2l+1} & \text{if } l = m \\ 0 & \text{otherwise} \end{cases}$$