(Note - I covered Griffith's 6.1.1, 6.1.2, + 6.1.4 already.

Magnetic fields affect matter, in 2 ways, one simple + obvious, the other ... less so!

(e.g. from electron's spin)

① If the matter has little dipoles in it $\vec{m}$ we saw in ch. 5

then they will tend to line up $\vec{B}_{ext}$ $\vec{m}$ $\vec{B}_{internal}$ Notes (P. 5-46)

Thus, the B field in matter tends to get stronger ($\vec{B}_{internal}$ from $\vec{m}$ lines up w. $\vec{B}_{ext}$)

[This is different from $\vec{E}$, where E_{ext} $\vec{p}$ $E_{internal}$

$\vec{p}$ also lines up, but that makes an

internal $\vec{E}$ field opposing (weakening) $\vec{E}_{ext}$]

This effect is call $\boxed{\text{paramagnetism}}$ - the $\vec{B}_{int}$ is parallel to $\vec{B}_{ext}$

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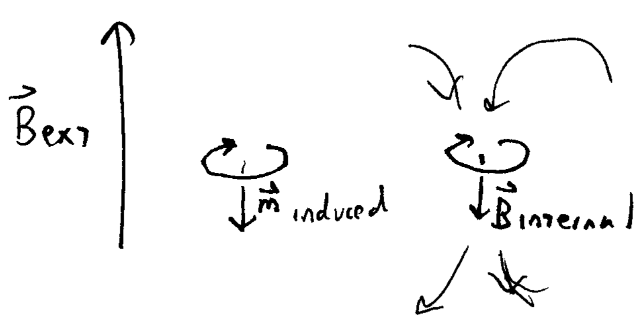
② Here's the other thing that can happen. The $\vec{B}_{int}$ can point the other way, diametrically opposed to $\vec{B}_{ext}$.

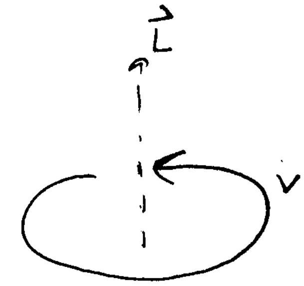
This is DIAMAGNETISM.



- Typically weaker than paramag
- It observed only if paramagnetism is absent, e.g. no little dipole moments or even # of electrons which tend to "anti-align" from QM.

Where does diamagnetism come from?

<u>Real</u> answer is Q.M. (!) Classical explanation is a cheat, but helps "make sense". So consider it a way to help visualize, without taking it too literally.



Consider e^- in <u>orbit</u> : $V \cdot T = d = 2\pi R$

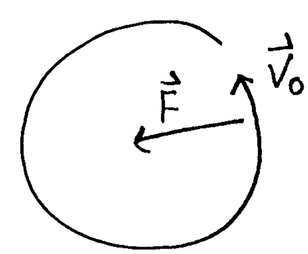
$$\text{so } I = \frac{\text{charge}}{\text{time}} = \frac{e}{T} = \frac{eV}{2\pi R}$$

Meanwhile magnetic dipole moment $m = I \text{ Area} = \frac{eV}{2\pi R} \cdot \pi R^2 = \frac{eVR}{2}$

And Angular momentum $L = m_e V R$ (m_e = mass of electron, m = mag moment)

Note: because of − charge, $\vec{m} = -\frac{e}{2m_e}\vec{L}$ (opp. direction)

So orbital ang momentum contributes to $\vec{m}$ of an electron (there's also m from "spin")



If e^- is in orbit, $\vec{F} = m_e \vec{a}$

$$\frac{e^2}{4\pi\varepsilon_0 R^2} = m_e \cdot \frac{V_0^2}{R}$$

Now slap on B field in $\hat{z}$ direction

<u>new</u> $\vec{F} = \frac{e^2}{4\pi\varepsilon_0 R^2} + ev'B$

- R stays same (! See next p.)
- V <u>changes</u> to v'

6-3

As B turns on, $dB/dt \neq 0$, so there is an <u>induced</u> E field (Faraday's Law) running in circles. It changes KE of electron, + turns out to be "just so" that R is unchanged, but V changes

$$\underbrace{\frac{e^2}{4\pi\epsilon_0 R^2}}_{= \frac{mv_0^2}{R}} + ev'B = \frac{m_e v'^2}{R}$$

so $\frac{m_e}{R}\left(v'^2 - v_0^2\right) = ev'B$

If B isn't huge,

$$v'^2 - v_0^2 = (v'-v_0)(v'+v_0)$$
$$\simeq \delta v \cdot \bar{v} \cdot 2 \simeq \delta v \, v' \cdot 2$$

so $\delta v \simeq \frac{eBR}{2m_e}$

But remember, $m = \frac{evR}{2}$, so $\delta m = \frac{eR}{2} \cdot \frac{eBR}{2m_e} = \frac{e^2R^2}{4m_e} B$

If $\vec{B}$ was in $+z$, $\Rightarrow$ speeds up, $\Rightarrow$ m is bigger but recall in <u>opposite</u> direction due to $-q$, so

$$\boxed{\delta \vec{m} = -\frac{e^2R^2}{4m_e}\vec{B}}$$

[This is diamagnetism, add $\vec{B}$ + get $\vec{m}$ changing in <u>opposite</u> way.

(If $\vec{B}$ was in $-z$ $\Rightarrow$ slows down $\Rightarrow$ $|m|$ gets smaller, $\delta\vec{m}$ is up, <u>still</u> in $-\vec{B}$ direction)

Bottom line:

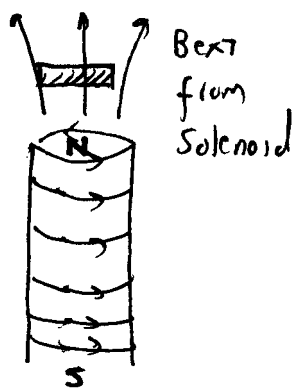
If material has permanent dipole moments (e.g. odd # of electrons, though other mechanisms are possible!)

then paramagnetism dominates.

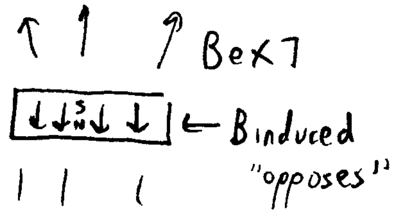
If $\vec{m}_{atoms} = 0$, likely to be diamagnetic.

If material is magnetized, $M = \frac{\text{mag dipole moment}}{\text{Volume}} \neq 0$

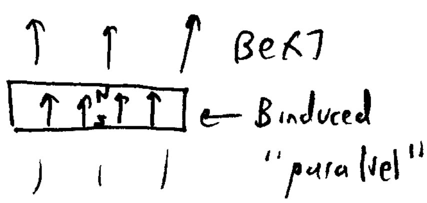
Might happen spontaneously, or because of $\vec{B}_{ext}$ ...



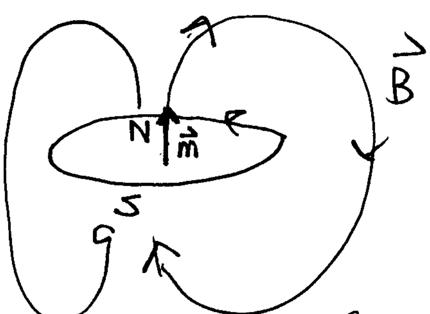
If material is diamagnet then it repels from Bext.



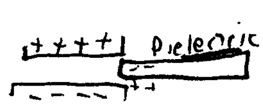
If material is paramagnet then it attracts into Bext



Remember



Both effects are very small, this is not "kitchen magnet" kind of story, that's Ferro magnetism!

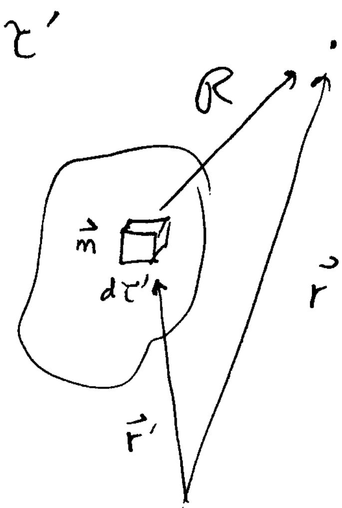
(Also, recall 

is attracted!)

Just as we did in ch. 4 for $\vec{E}$ from many electric dipoles, we can find $\vec{B}$ from many magnetic dipoles.

And, just as we used Voltage to do this in ch. 4 (Griff 4.2.1) we'll use $\vec{A}$ here.

$$\vec{A}_{\text{ideal magnetic dipole}} = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\hat{\mathscr{R}}}{\mathscr{R}^2}, \quad \text{+ when } \vec{m} = \vec{M}\, d\tau'$$



$$\text{so} \quad \vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\iiint \frac{\vec{M}(r')\times\hat{\mathscr{R}}}{\mathscr{R}^2}\, d\tau'$$

In principle, that's it! But as before, we can massage this to get a much more useful + intuitive result: [Quick proof, see Griffiths p. 264]

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\oiint_{\text{surf}} \frac{\vec{K}_{\text{bound}}(\vec{r}')}{\mathscr{R}}\, da' + \frac{\mu_0}{4\pi}\iiint_{\text{volume}} \frac{\vec{J}_{\text{bound}}(r')\, d\tau'}{\mathscr{R}}$$

The mag potential due to ordinary surface current

mag (vector) potential due to ordinary current density

with $\boxed{\vec{K}_b = \vec{M}\times\hat{n} \quad \text{and} \quad \vec{J}_B = \vec{\nabla}\times\vec{M}}$

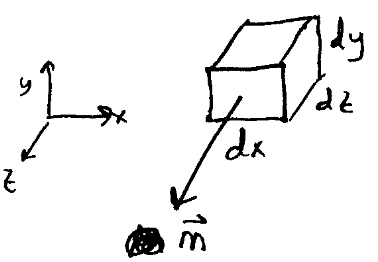
see my notes p. 4-3 for close Electric analogues!

(recall) $\sigma_b = \vec{P}\cdot\hat{n}$ and $\rho_B = -\vec{\nabla}\cdot\vec{P} \longleftrightarrow$

So there are these "imaginary" or "effective" bound currents; a magnetically polarized object <u>acts</u> like it had ordinary currents flowing in it (+ then what we just learned about $\vec{B}$ field arising from such currents tells us resulting B field)

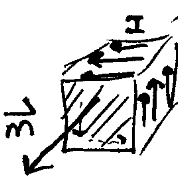
Let's think about the meaning / physics of these 2 terms!

Consider 1st a small chunk of material with $\underline{M} = \frac{m}{dx\,dy\,dz}$



Say $\vec{M}$ points in $+\hat{z}$ direction

This would <u>arise</u> from current running around like this:



really, some $\underline{K}$ on each of 4 side walls

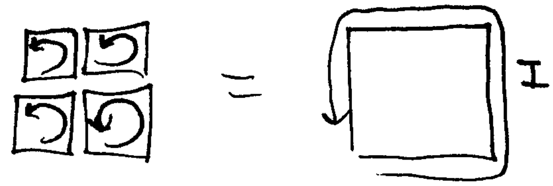
Now $m = I \cdot \text{Area} = (K \cdot l_\perp)\,\text{area}$. On <u>this</u> cube, $l_\perp = dz$, area $= dx\,dy$

so $m = K\,dx\,dy\,dz$

thus $\underline{M} = K$.

Direction of K? Look at picture, on <u>each</u> wall, it's $\underline{M} \times \hat{n}$.

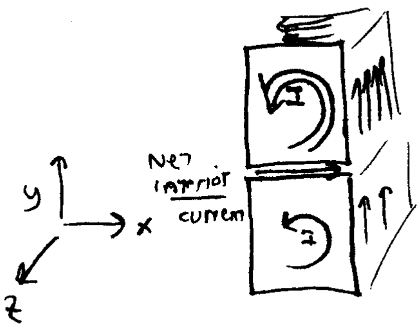
If $\vec{M}$ is constant, then it can be thought of as arising just from <u>surface</u> current (interior currents cancel!)



Funny: [Solid body with <u>tiny</u> atomic currents <u>acts</u> like there was a "macro" current running all way around outside.

6-7

What if $\vec{M}$ is not constant? Consider e.g. $\vec{M}$ points in $\hat{z}$, but Bigger in top cell than in bottom cell.



Now "internal" currents do not cancel, there is a $\vec{J}$ inside.

In this case, $I_{interior} = (J_B)_x\, dA_\perp$

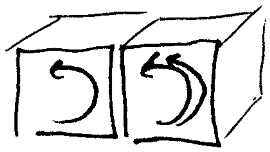
$$= (J_B)_x\, dy\, dz \quad \leftarrow \text{Do you see why?}$$

But $I_{interior} = I_{upper} - I_{lower}$, (+ use $M = K = I/dl_\perp = I/dz$)

$$= M\left(y + \frac{\delta y}{2}\right) dz - M\left(y - \frac{\delta y}{2}\right) dz$$

$$= \frac{\partial M}{\partial y}\, \delta y\, dz$$

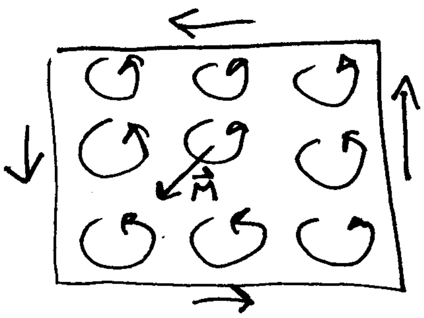
so $(\vec{J}_B)_x = \dfrac{\partial M}{\partial y}$ in this case, (with M purely $M_z \hat{z}$)

of course, if we'd considered 

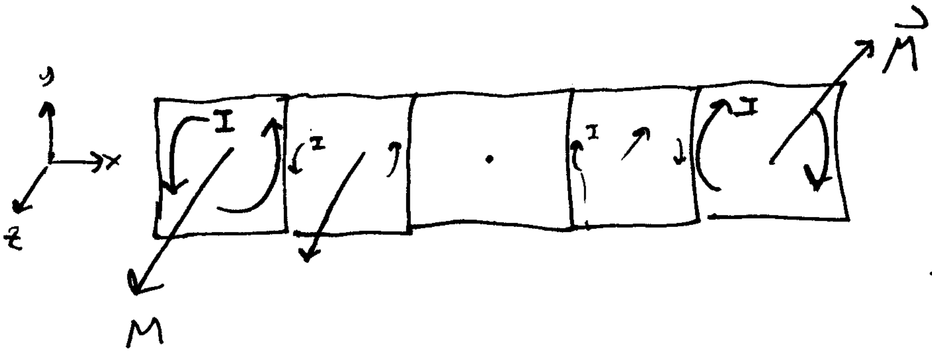
we'd get $(\vec{J}_B)_y = -\dfrac{\partial M}{\partial x}$

But $\vec{\nabla} \times \vec{M} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial_x & \partial_y & \partial_z \\ 0 & 0 & M \end{vmatrix} = \dfrac{\partial M}{\partial y}\hat{x} - \dfrac{\partial M}{\partial x}\hat{y}$

so we're getting $\vec{J}_B = \vec{\nabla} \times \vec{M}$

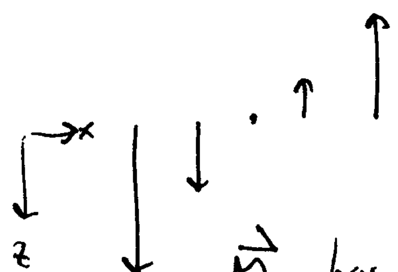


$\vec{K} = \vec{M} \times \hat{n}$ gives surface current around <u>outside</u> of cell.



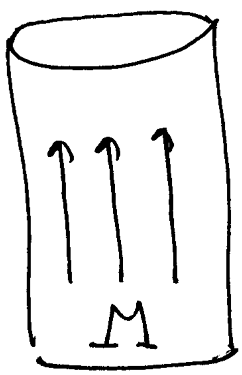
Here we see net current flowing <u>inside</u> too, In this example <u>up</u> page everywhere. (~~...~~ Direction of $\vec{\nabla} \times \vec{M}$, do you see that?)

Looking down from +y :



$\vec{M}$ has curl out of page.

<u>Example</u>



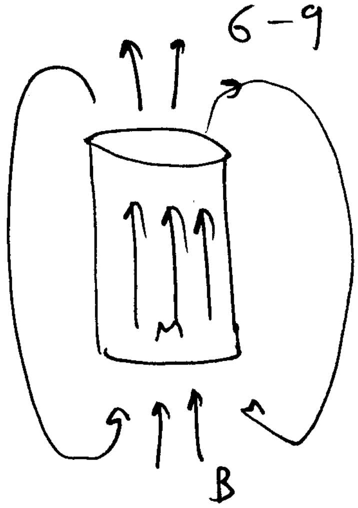
Cylinder, uniformly magnetically polarized

Inside, $\vec{\nabla} \times \vec{M} = 0$ (uniform!)

so no bound <u>currents</u>

~~...~~ Also, $\underline{M} \times \hat{n} = 0$ on top/bottom

But, on walls, $\vec{K} = \vec{M} \times \hat{n} = M\hat{\varphi}$ uniform "circulating" surface current. So, this is like a (finite) Solenoid!

so 

This is a permanent magnet, which looks just like a finite solenoid (inside and out) even though there is no "wire" with current wrapped around it.

(Bunch of tiny atom currents conspiring!)

Example: Uniformly magnetized sphere 

again, $\vec{J}_B = 0$

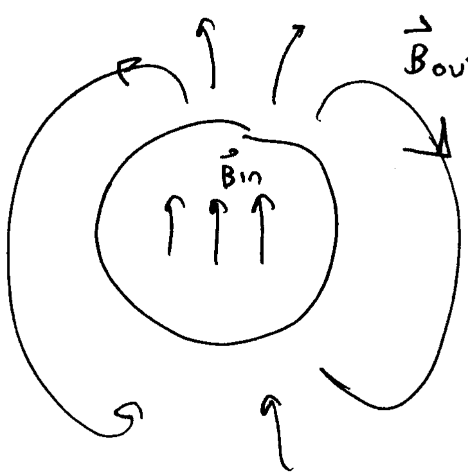
But $\vec{K} = \vec{M} \times \hat{n} = M \hat{z} \times \hat{r} = M \hat{\varphi} \sin\theta$

Griffiths solved for just such a problem in Ex 5.11

(also my notes p. 5-37-38)

He found $\vec{B}_{in}$ was uniform $\frac{2}{3}\mu_0 R \vec{M}$

$\vec{B}_{out}$ is a perfect dipole field, (with $\vec{m} = \vec{M} \cdot \frac{4}{3}\pi R^3$ of course!)



In general, inside any material, you'll have free currents (basically, wires running through it, or flowing ions...) and, as a result, $\vec{B}$ fields appear which further polarize the material, adding in these bound currents. (which in turn alter the field even more!)

All together, in "equilibrium"

$$\vec{J} = \vec{J}_{free} + \vec{J}_{bound}$$

what you "inject"

what the material responds with

This $\vec{J}$ is real, it creates the total, real $\vec{B}$ field via Ampere's law, $\oint \vec{B}\cdot d\vec{l} = \mu_0 \iint \vec{J}\cdot d\vec{A}$

$$\underbrace{\vec{\nabla}\times\vec{B} = \mu_0 \vec{J}} = \mu_0\left(\vec{J}_f + \vec{J}_{bound}\right) = \mu_0\left(J_f + \vec{\nabla}\times\vec{M}\right)$$

no exceptions in magnetostatics, this is Ampere's law

[See notes p.4-11 the Electric story was very similar, leading to $\vec{D}$]

so $\vec{\nabla}\times\left(\dfrac{\vec{B}}{\mu_0} - M\right) = \vec{J}_f$

so we Define $\vec{H} = \dfrac{\vec{B}}{\mu_0} - \vec{M}$

with $\vec{\nabla}\times\vec{H} = \vec{J}_f$ or $\oint \vec{H}\cdot d\vec{l} = I_{f,\text{through}}$

(usually easy to measure, this is the current in wires, normally!)

[Units of H are $\frac{\text{Amps}}{m}$, not Teslas!]

[Not really sure what to call H, it's just H!]

Example: