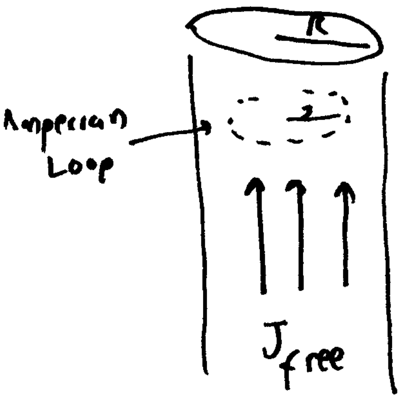
<u>Example</u> A long <u>Al</u> rod (Radius R) carries <u>uniform</u> J_{free}, (total current $I = J_f \cdot \pi R^2$) in $+z$ direction.

Find H and B everywhere.

Like Griff Ex 6.2, but Al is <u>Paramagnet</u>, <u>his</u> example was Cu = <u>dia</u>magnet, so <u>compare</u> this with his example!



I <u>expect</u> $\vec{B} = \circlearrowleft$ by Ampere's law

(just like always, current is <u>up</u>, $\vec{B}$ circulates)

I expect $\vec{M}$ <u>inside</u> will be parallel to $\vec{B}$ because it's a <u>para</u>magnet so $\vec{M} = \circlearrowleft$ too

of course, <u>outside</u> is vacuum, so $M_{OUTSIDE} = 0$.

Since $\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$, these "cancel", but I know $\vec{M}$ is going to be small / weak for real materials, so I'm sure $\vec{H}$ will still (also) go $\circlearrowleft$ this way. Indeed,

$\oint \vec{H} \cdot d\vec{l} = I_{f,\,through}$ <u>proves</u> that $\vec{H}$ "looks" like $\vec{B}$,

Loop <u>inside</u>, shown dashed in direction at least

So $\oint H \cdot dl = I_{f,\text{through}} \Rightarrow H \cdot 2\pi s = J_f \cdot \pi s^2$

$$\vec{H} = \frac{J_f s}{2}\hat{\varphi} = \frac{I}{2\pi R^2} s \hat{\varphi} \qquad \left(\text{since } I = J_f/\pi R^2\right)$$

this is same as Griff, dia or para, makes no diff!

Outside $\vec{H} = \frac{I}{2\pi s}\hat{\varphi}$ (same as Griff, doesn't matter what material is)

outside $\vec{M} = 0$, so $\vec{B} = \mu_0 \vec{H} = \frac{\mu_0 I}{2\pi s}\hat{\varphi}$ ← usual old infinite wire result, material independent!

Inside, we know direction of $\vec{M}$, + know it's less than $\frac{\vec{B}}{\mu_0}$, but we're stuck without knowing how it magnetizes. We'll get to that soon!

But I can argue qualitatively that if $\vec{M}$ looks like ↻

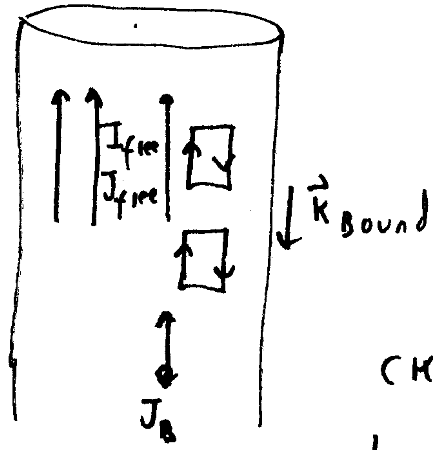
then $\vec{M} \times \hat{n}$ will run down the outside of cylinder (parallel to J_f)

(that's opposite Griff example!)

and $\vec{\nabla} \times \vec{M} = J_B$ points up the cylinder (~~opposing~~ parallel to J_f,

which is consistent with para magnetism, (and opposite Griff ex)

So



But to compute $\vec{J}_B$ and $\vec{K}_B$, we need $\vec{M}$ inside, and I can't get that from my (known) $\vec{H}$ inside without knowing how Al magnetizes. So...

Linear Materials

Many (common) materials magnetize proportional to $\vec{B}$

Recall electric polarization was $\vec{P} = \varepsilon_0 \chi_E \vec{E}_{TOT}$ (linear dielectrics)

we define χ_M for linear magnetic materials as

$$\vec{M} = \chi_M \vec{H}$$

note the lack of "symmetry", in the case of dielectrics, χ_E is defined looking at $\vec{E}$, but for magnetic materials, we use $\vec{H}$, not $\vec{B}$. Why? Because $\vec{H}$ is easy to compute in many cases! $\oint \vec{H} \cdot d\vec{l} = I_{free}$, ← that's what you measure + control, not $I_f + I_b$

χ_M is "magnetic susceptibility"
Unitless, typically small.

χ_M is $+$ for paramagnets, $\vec{M}$ lines up with $\vec{H}$

χ_M is $-$ for diamagnets, $\vec{M}$ opposes $\vec{H}$.

of course, $\vec{H} = \dfrac{\vec{B}}{\mu_0} - \vec{M}$ so $\vec{B} = \mu_0(\vec{H} + \vec{M})$

$$= \mu_0 \vec{H}\,(1 + \chi_M)$$

so $\vec{H}$ and $\vec{B}$ point in the same direction, if $\chi_M > 0$

and even if $\chi_M < 0$, as long as it's < 1.

For normal materials, $|\chi_M|$ is like 10^{-5} to 10^{-9}

For superconductors, $\chi_M = -1$. (which means $\vec{B} = 0$ inside, total "shielding")

Summary

$$\boxed{\begin{aligned} \vec{M} &= \chi_M \vec{H} \\ \vec{B} &= \mu_0(1+\chi_M)\,\vec{H} \equiv \mu \vec{H} \\ \vec{M} &= \frac{\chi_M}{\mu}\,\vec{B} \end{aligned}}$$

"permeability"

Note: free space $\vec{B} = \mu_0 \vec{H}$, so $\mu_0 = 4\pi \cdot 10^{-7}\ \frac{T\,m}{A}$ is "permeability of free space"

Back to our Al rod example: $\chi_{Al} = +2\cdot 10^{-5}$

$\uparrow$ Paramagnetic

we had found (p. 6-12) $\vec{H}_{inside} = \frac{I}{2\pi R^2} s \hat{\varphi}$

so $\vec{M}_{inside} = \chi_M \frac{I}{2\pi R^2} s \hat{\varphi}$ very small

$\vec{B}_{inside} = \mu \frac{I}{2\pi R^2} s \hat{\varphi}$ which is almost identical to

what we got in ch. 5 without knowing about magnetization,

since $\mu = \mu_0(1+\chi_M)$. However, $\mu > \mu_0$ (a bit)

$\uparrow$ tiny! so $\vec{B}_{inside}$ is "enhanced" a bit.

That's paramagnetism. A copper wire has $\chi_M \approx -10^{-5}$,

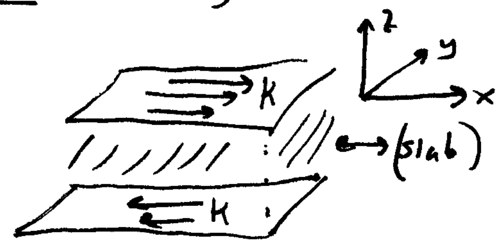
so $\vec{B}_{inside}$ is ever so slightly reduced

(outside, none of this matters)

Example: TWO ∞ SHEETS CARRY CURRENT I,

TOP in +x direction

Bottom in −x "

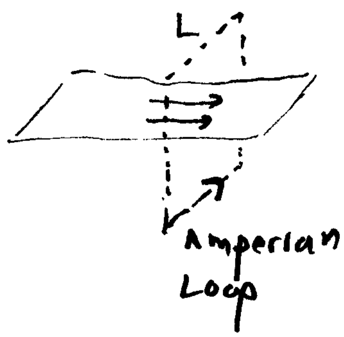


Both are same $|K|$.

Between sheets is "linear material" (slab with susceptibility χ_m)

what's $\vec{B}$, $\vec{H}$, $\vec{M}$ in slab? First, $\oint H\cdot d\ell = I_{free}$ ← that's what "K" gives!

By symmetry, I know $\vec{H}$ will point into page ($+\hat{y}$) between slabs, + will cancel to zero outside (Think back to this same example in free space with no slab!)



so this AMPERIAN LOOP gives $\oint H\cdot d\ell = H\cdot L$

$I_{free} = K\cdot L$

so $\vec{H} = K\hat{y}$ in between.

$= 0$ outside

so $\vec{M}_{inside} = \chi_m \vec{H} = \chi_m K\hat{y}$

$$\vec{B}_{inside} = \mu\vec{H} = \mu K\hat{y} = \mu_0(1+\chi_m)K\hat{y}$$

Pretty much same as free space, just slight ↑ enhancement for paramag reduction for diamag.

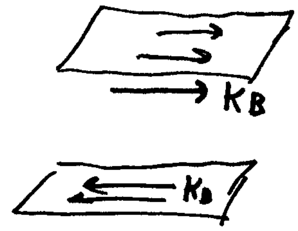
What do Bound currents look like in that example?

inside $\vec{\nabla} \times \vec{M} = 0$ (since $\vec{M}$ is uniform)

so, no Bound $\vec{J}_B$.

But $\vec{M} \times \hat{n} = +\chi_m K \hat{x}$ at top so

$-\chi_m K \hat{x}$ at bottom



this is parallel to K_{free}, but very small, if $\chi_m > 0$

Makes sense, that's the mechanism to enhance $\vec{B}$ inside

if $\chi_m < 0$, this opposes K_{free}, reducing $\vec{B}$ inside a little.

Boundary conditions

Since $\oint H \cdot dl = I_{free}$, $H_{\parallel}$ can "jump" at boundaries.

$$H''_{above} \cdot L - H''_{Below} \cdot L = K_f \cdot L$$

or, in vector notation (since H'' in fact has two possible components we should really write

$$\vec{H}''_{above} - \vec{H}''_{below} = \vec{K}_f \times \hat{n}$$

convince yourself this gives right answer for "sheet" example above!

Meanwhile, $\vec{H} = \frac{1}{\mu_0}\vec{B} - \vec{M}$

so $\vec{\nabla}\cdot\vec{H} = -\vec{\nabla}\cdot\vec{M}$ which says

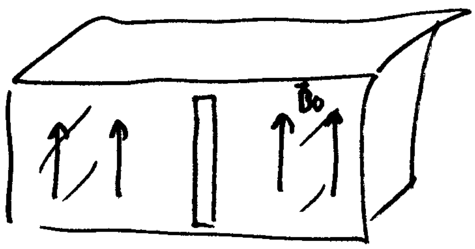
$$H_\perp^{above} - H_\perp^{below} = -\left(M_\perp^{above} - M_\perp^{below}\right)$$

This will vanish if $\vec{M}$ is continuous. Since $\vec{M} = \frac{\chi_m}{\mu}\vec{B}$ for linear materials, and $\vec{B}_\perp$ is always continuous by $\vec{\nabla}\cdot\vec{B} = 0$, it means $H_\perp$ is always continuous everywhere except where χ_m suddenly changes (edge of a material)

Consequence: $\oint \vec{H}\cdot d\vec{l} = I_{free}$ looks simple, and for cases of "high symmetry" lets you deduce $\vec{H}$ easily (a la Ampere's law). But if symmetry is not high, beware.

E.g. if $I_{free} = 0$ everywhere, you cannot conclude $H = 0$ everywhere! Think of a toy magnet! Just because $\vec{\nabla}\times\vec{H} = 0$ everywhere does not mean $\vec{H} = 0$ everywhere (unless some symmetry argument can be invoked, like our infinite sheet example)

Example:



Big chunk of material with uniform $\vec{B}_0$ (up) throughout.

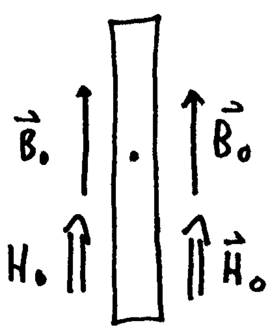
That $\vec{B}_0$ is the total $\vec{B}_0$ field, arising (perhaps) from external $\vec{B}$ and the magnetization of the material, superposed.

so that material has $\vec{M}_0 = \dfrac{\chi_m}{\mu} \vec{B}_0$ (uniform)

$\vec{H}_0 = \dfrac{1}{\mu} \vec{B}_0$ (uniform, up) no matter what χ is, $\mu > 0$.

[and $\vec{H}_0 = \dfrac{\vec{B}_0}{\mu_0} - \vec{M}_0$, of course]

Then I dig a "needle shaped" hole, as shown. What's $\vec{B}, \vec{H}$ inside the hole? (clearly $\vec{M} = 0$ in there, it's a hole!)
(at the center)



There are no free currents at this boundary,

so $H^{\parallel}_{above} - H^{\parallel}_{below} = 0$

which says $H = H_0 \hat{z}$ inside the hole

But that means $\vec{B} = \mu_0 \vec{H} = \mu_0 H_0 \hat{z}$ inside the hole, at center.

In terms of B_0 + M_0, this is $\mu_0 H_0 = B_0 - \mu_0 M_0 = \dfrac{B_0}{1+\chi_M}$

so $\vec{B}$ is reduced in there, if $\chi_M > 0$
$\vec{B}$ is enhanced if $\chi_M < 0$.

Due to Bound currents on walls, presumably! Picture it! Like solenoid...

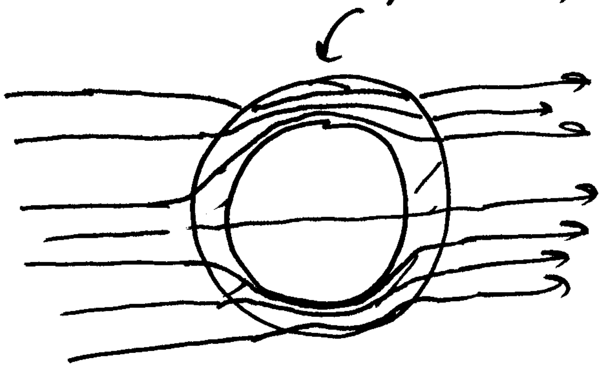
"μ METAL" has high χ_M, shields B inside!

6.19 supp.

"μ metal" = $Ni_{0.77}Fe_{0.16}Cu_{0.05}Cr_{0.02}$ has

$\frac{\mu}{\mu_0} = 10^5$ ($\chi = +10^5$, unlike most paramags with typical $\chi \simeq 10^{-3}$)

inside μ-metal, B likes to be high, it's a super para magnet.

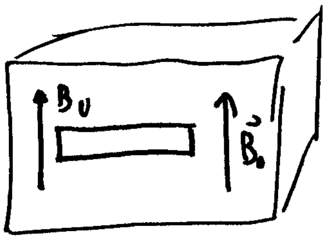


Inside the "hole", B is quite weak.

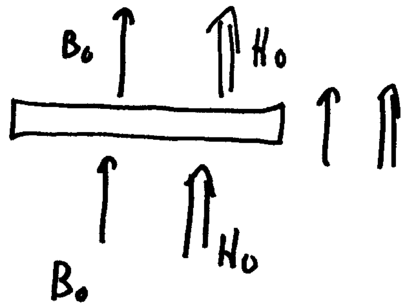
This is used on equipment where you want to eliminate external $\vec{B}$ fields passively.

6-20

Let's do that for a wafer-like cavity:



now



This time, I can argue $B_\perp^{above} = B_\perp^{below}$, so $\vec{B} = B_0\hat{z}$ at center

so this time $\vec{B}$ is not modified!

But then $\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M} = \frac{B_0}{\mu_0}\hat{z}$ is not H_0!! It's $\frac{\mu}{\mu_0}H_0$!

so $\vec{H}_{center} = \frac{\mu}{\mu_0}H_0 = \frac{B_0}{\mu_0} = H_0 + M_0$

for paramag material, H_{center} is enhanced $(\mu > 1)$
dia " H_{center} is reduced. $(\mu < 1)$

(remember, $H_\perp$ can jump at boundary, if $M_\perp$ suddenly changes!)

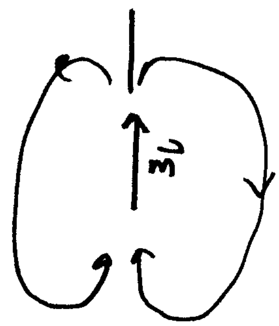
Here, it's like we've "superposed" a disk of $\boxed{\downarrow\downarrow\downarrow\downarrow\ M_0}$

$B^{superposed}_{center} \sim \frac{1}{R}$ will be small if disk has big R, hence $\vec{B}$ unchanged.

In prev example $\downarrow\downarrow\downarrow$ we superposed a solenoid which does change $\vec{B}$ inside!

6-21

Ferromagnetism : won't spend a ton of time on this, it's pretty complicated microscopically, but some materials have a unique property, due to QM electron-pair interactions, that favors spin aligned electrons.



I can easily see why spins above & below like to align, but not why spins on sides would. It's not a classical effect!

This means $\vec{M}$ "locks in", it's not a response to $\vec{B}_{ext}$, so you cannot define χ_m or μ for ferromagnet. It's not a linear material!

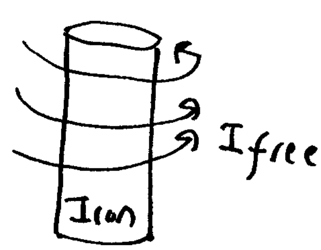
It's a local effect, & you find regions of parallel spins $\Rightarrow \vec{m}$, called domains. See fig 6.26 in Griffiths. In your fridge, you can align those domains in presence of $\vec{B}_{ext}$ (kitchen magnet) making strong para like effect $\Rightarrow$ attraction.

when pull magnet away, thermal relaxation randomizes domains quickly, so fridge doesn't remain magnetized. ("Magnet" does $\Rightarrow$ it's a very special ferromagnet!)

6-22.

- Fe, Co, + Ni are about the only materials like this
- At high T, even permanent mags will randomize ("Curie Temp" is the critical value, above which they "melt their alignment"
- Typical Fe domains are ~ mm on a side.
- " " Magnetization in a domain is $M \sim 10^6 \frac{A}{m}$ which yields typical $\underbrace{\vec{B} = \frac{2}{3}\mu_0 \vec{M}}_{\text{for spherical domain}} \sim \underline{\underline{1\,T}}$. So that's about the most you're likely to get from normal magnets
- Applying $\vec{B}$ to ferromagnet merely re-aligns domains. So small $\vec{B}_{ext}$ can radically (non-linearly!) alter $\vec{B}_{tot}$.

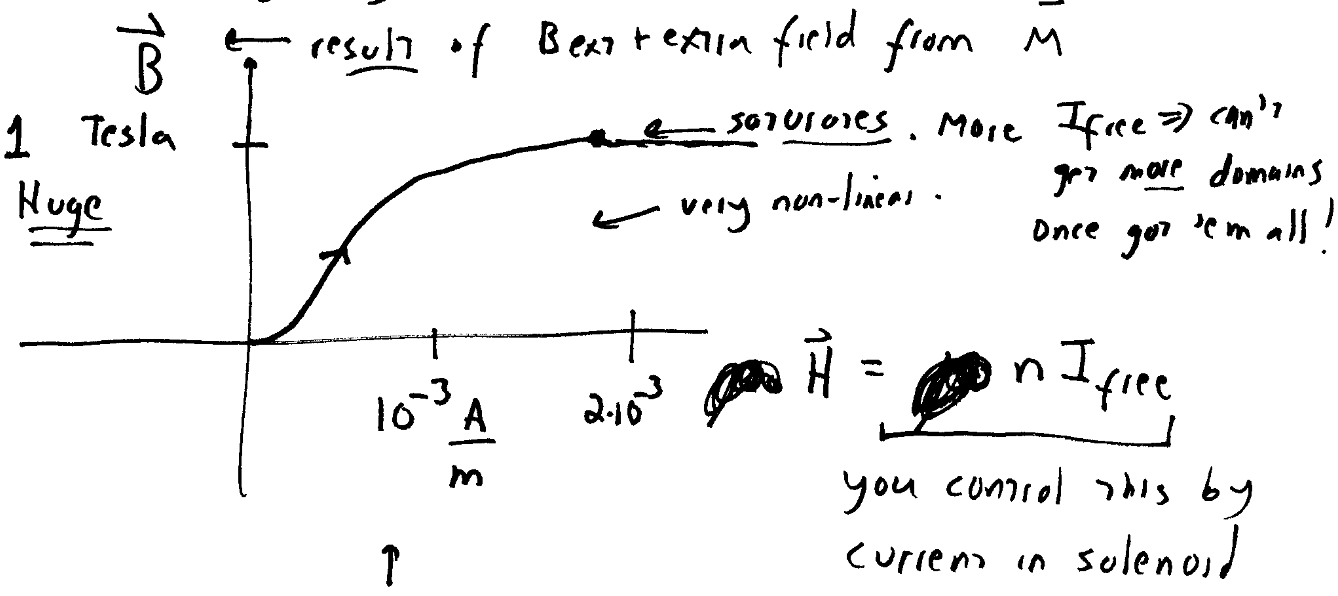
For Metal inside Solenoid (Iron, e.g)



the $I_{free} \Rightarrow \vec{H}_{inside}$ in usual way, by Ampere: $\vec{H}_{inside} = n\vec{I}_{free}$

But $\vec{B}_{inside}$ will be $\mu_0(\vec{H} + \vec{M})$, + M can be huge (+ depends on $\vec{H}$, more field $\Rightarrow$ more domains join in, influencing each other too)

6-23



Note that, by itself, this solenoid would only generate

$$\vec{B}_{\substack{\text{vacuum}\\ \text{solenoid}}} = \mu_0 \vec{H} = 4\pi \cdot 10^{-7} \frac{T\,m}{A} \times 10^{-3} \frac{A}{m} \simeq 4\pi \cdot 10^{-10}\, T$$

so we're ~~go~~ getting billion fold enhancement!

If you back off the current, B decreases, but some alignment remains! This is "hysteresis", memory.