P3310 LECTURE 8 (topic continued from Lecture 7) ENERGY IN THE ELECTROSTATIC SY[illegible]

Take $\rho, V \Rightarrow E$:

$$W = \frac{\varepsilon_0}{2}\int d^3r\,(\vec{\nabla}\cdot\vec{E})V$$

$$= \frac{\varepsilon_0}{2}\left[-\int d^3r\,\vec{E}\cdot\vec{\nabla}V + \oint d\vec{A}\cdot V(\vec{r})\vec{E}\right.$$

replace $E \Leftrightarrow -\vec{\nabla}V$:

$$W = \frac{\varepsilon_0}{2}\left(\int_V d^3r\,E^2 + \oint_S V\vec{E}\cdot d\vec{A}\right)$$

And enlarge surface to include all space $\rightarrow V = 0$,

$$W = \frac{\varepsilon_0}{2}\int \vec{E}^2\,d^3r$$

Check: is this consistent with discrete case with $\rho(\vec{r}) = \delta^3(\vec{r})Q$?

...this is an apparent contradiction, since if you take the energy of a collection of point charges (or even just one!) you get infinity!

Where did we go wrong? Discrete case: $W = \frac{1}{8\pi\varepsilon_0}\sum_{i=1}^{N}\sum_{j=1 \neq i}^{N}\frac{Q_i Q_j}{r_{ij}}$

The $j \neq i$ criterion in the sum is effectively dropped when going to the continuous integral. Doesn't matter, since the charge at any given point is zero in a continuous distribution. But you then cannot take $\rho(\vec{r})$ as a bunch of 3D δ-functions and expect to get the answer from previous lecture.

Note — since energy is $\frac{\varepsilon_0}{2}\int d^3r\, E^2$, the energy from systems whose fields overlap does not just add. (Griffiths: "no superposition" for energy). This should be obvious for a couple of reasons:

- Two charges have fields whose individual integrals don't change with charges' location: but energy of the two-charge (or two-distribution) system depends on the relative location of the charges!
- $E^2 = E_1^2 + E_2^2 + 2\vec{E}_1 \cdot \vec{E}_2$
  $\neq E_1^2 + E_2^2$.

Where is the energy? It's really in the field! The charges are dumb — they only respond to the local field.

<u>Conductors</u> — materials where charge is free to move.

Characteristics follow from that statement:

- $\vec{E} = 0$ inside a conductor
- $\rho = 0$ follows from $\vec{E} = 0$
- All nonzero charge density is surface charge
- Conductor is equipotential
- $\vec{E}$ is perpendicular to the surface