Electromagnetic Units: this area has many competing standards! Three basic choices to make:

- choice of mechanical units (cgs, mks, foot-pound ...)
- choice of charge definition (where c shows up)
- "rationalized" or not (coupled to above) — where the 4π shows up. (The 4π is a volume integration factor.)

In principle, could mix and match from all these choices. In practice, only three systems are in common use:

SI = "rationalized mks". Mechanical units are mks, and the 4π shows up in the denominator of Coulomb's Law: $\vec{E} = \frac{1}{4\pi\varepsilon_0 r^2} Q\hat{r}$
Unfortunately, charge defined in a way that causes factor of c to be "split" into ε_0 and μ_0 — so one of these shows up everywhere. Another consequence is that $\vec{E}$ and $\vec{B}$ have different units, $\vec{E}$ and $\vec{D}$, $\vec{B}$, and $\vec{H}$ too: $\frac{E}{B}$ has velocity units.

Gaussian: unrationalized cgs: Mechanical units are cgs:

$$\text{force} = \frac{g\,cm}{s^2} = \text{dyne } (10^{-5}\,N)$$
$$\text{energy} = \frac{g\,cm^2}{s^2} = \text{erg } (10^{-7}\,J)$$

Unrationalized: 4π is in numerator in source terms:
$\operatorname{div} \vec{E} = 4\pi\rho$, but $\vec{E} = \frac{1}{r^2} Q\hat{r}$.

Charge definition: the "c" shows up explicitly in magnetic quantities; no separate ε_0 or μ_0 factors. Consequence is that $\vec{E}, \vec{B}, \vec{D}, \vec{H}$ all have same dimensions!

Major equations take following forms. Start with basic electrostatic field equations:

MKS (SI)	Gaussian
$\operatorname{div}\vec{E} = \varepsilon_0 \rho$	$= 4\pi\rho$
$\operatorname{div}\vec{D} = \rho_f$	$= 4\pi\rho_f \implies \vec{D} = \vec{E} + 4\pi\vec{P}$
$\operatorname{curl}\vec{B} = \mu_0 \vec{J}$	$= \frac{4\pi}{c}\vec{J}$
$\operatorname{curl}\vec{H} = \vec{J}_f$	$= \frac{4\pi}{c}\vec{J}_f \implies \vec{H} = \vec{B} - 4\pi\vec{M}$

Heaviside-Lorentz units: Rationalized version of Gaussian. Not common except in certain theoretical fields:

$$\operatorname{div}\vec{E} = \rho, \quad \operatorname{curl}\vec{B} = \frac{1}{c}\vec{J}.$$

Force laws: $\vec{F} = Q\vec{E}$ in all systems

$$\vec{E} = \frac{Q\,\hat{r}}{r^2} \quad \text{in Gaussian}$$

$$= \frac{Q\hat{r}}{4\pi\epsilon_0 r^2} \quad \text{in SI}$$

$$= \frac{Q\hat{r}}{4\pi r^2}$$ in H-L: Note the clear geometrical interpretation —

$$|E| = \frac{Q}{A}$$ where A = area of the sphere.

Magnetic force law:

$$\text{SI}: \quad \vec{F} = Q\vec{v}\times\vec{B}$$

$$\text{Gauss}: \quad \vec{F} = Q\frac{\vec{v}}{c}\times\vec{B} \implies \frac{v}{c} \text{ is dimensionless.}$$

Biot - Savart:

$$\vec{B} = \frac{\mu_0}{4\pi}\int d^3r' \, \frac{\vec{J}\times\hat{r}}{r^2} \qquad \text{SI}$$

becomes $$\vec{B} = \frac{1}{c}\int d^3r' \, \frac{\vec{J}\times\hat{r}}{r^2} \qquad \text{Gaussian.}$$

Linear media behave nicely in Gaussian units:

$$\vec{D} = \varepsilon\vec{E} \quad , \quad \vec{B} = \mu\vec{H}$$

$$\hookrightarrow \varepsilon = 1 + 4\pi\chi_e \qquad \mu = 1 + 4\pi\chi_m$$

ε, μ are dimensionless. Comparing to SI:

$$\varepsilon_{gauss} = \frac{\varepsilon_{SI}}{\varepsilon_0} \quad , \quad \mu_{gauss} = \frac{\mu_{SI}}{\mu_0}$$

Energy density of field:

$$\text{SI}: \quad W = \int d^3r \left(\frac{\varepsilon_0}{2}E^2 + \frac{1}{2\mu_0}B^2\right)$$

$$\text{Gauss}: \quad = \frac{1}{8\pi}\int d^3r \left(E^2 + B^2\right)$$

$$\text{esu} = \frac{1}{3e9}\ \text{C}$$

$$\frac{\text{esu}}{\text{s}} = \frac{1}{3e9}\ \text{A}$$

$$\text{statvolt} = 300\ \text{V}$$

$$\text{gauss} = 10^{-4}\ \text{T} = 1\ \frac{\text{statvolt}}{\text{cm}} \text{ for } \vec{E}.$$