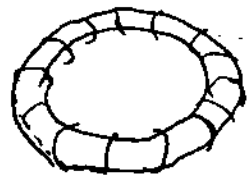
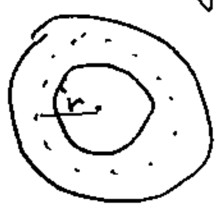
Continuing magnetic materials: Ferromagnetic toroids

Consider a toroidal piece of ferromagnetic material, with a current coil around it: N coils, current I:

What are boundary conditions on $\vec{B}$, $\vec{H}$?



Use Amperian loop along central toroidal axis:

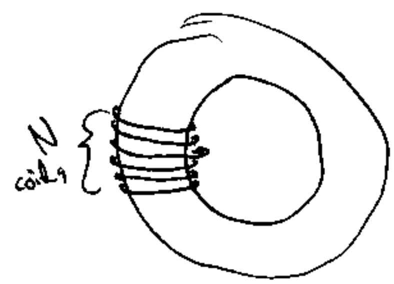


Symmetry allows calculation of H from I_{free}: $\oint d\vec{l}\cdot\vec{H} = I_{free} = N\underline{I}$

$\Rightarrow \vec{H} = \frac{NI}{2\pi r}\hat{\phi}$ and (assume linear): $\vec{B} = \mu\vec{H} = \frac{\mu NI}{2\pi r}\hat{\phi}$

Very straightforward. What if all the coils are in a small region of toroid?

Integrals of $\vec{B}$, $\vec{H}$ must be the same.



This is a "magnetic circuit": flux is confined inside the material, since there's no source of divergence or curl of $\vec{M}$, and $\vec{B}$ is **kept** parallel to surface.

$\Rightarrow \vec{B}$ is <u>same</u> as if coils were distributed around the ring!

$$\oint d\vec{l} \cdot \vec{H} = NI \quad \rightarrow \text{consider only } \phi \text{ component}$$

$$\oint dl \, H = NI$$

Now, express H as function of total flux Φ through a cross-section of the toroid:

$$\Phi = BA = \mu HA \Rightarrow H = \frac{\Phi}{\mu A}.$$

$$\oint dl \frac{\Phi}{\mu A} = NI$$

$\Longrightarrow$ In "contained flux" approx, take Φ constant along l.

$$\underset{\uparrow \atop \text{magnetic flux}}{\Phi} \underbrace{\oint dl \frac{1}{\mu A}}_{\text{Reluctance}} = \underbrace{NI}_{\text{Source current}}$$

$$\mathcal{R} = \frac{2\pi R}{\mu A} \text{ if simple toroid}$$

Note — often called "magnetic Ohm's Law:"

$$\Phi = \frac{NI}{\mathcal{R}}$$

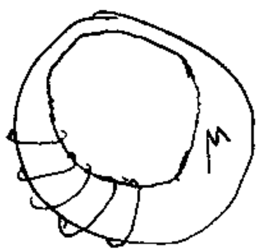
So flux plays role of current in Ohm's Law

permeability $\longleftrightarrow$ conductivity

reluctance $\longleftrightarrow$ resistance

NI $\longleftrightarrow$ voltage.

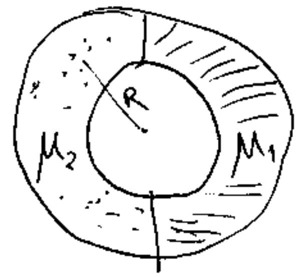
Can now also use this to predict flux with other geometries:



$$\mathcal{R} = \oint \frac{dl}{\mu A} = \frac{1}{\mu}\oint \frac{dl}{A}$$

A constriction in the toroid $\Longleftrightarrow$ increased reluctance
$\Longrightarrow \Phi$ decreases

Two materials:
(constant cross-sec.)



$$\mathcal{R} = \oint \frac{dl}{\mu A}$$

$$= \frac{1}{\mu_2 A}(\pi R) + \frac{1}{\mu_1 A}(\pi R) = \frac{\pi R}{A}\left(\frac{1}{\mu_1} + \frac{1}{\mu_2}\right)$$

Air gap: <u>be careful!</u> If gap is big, contained-flux assumption breaks down! <u>Narrow</u> gap: treat as above, with $\mu = \mu_0$.
($\mu_0 \approx \mu_{air}$ compared to ferromagnetism).

Can design many geometries of electromagnets this way.

dipoles: quadrupoles: